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**Essays on Inflation Targeting, Price Stability and  
The Conduct of Monetary Policy  
Under Imperfect Credibility**

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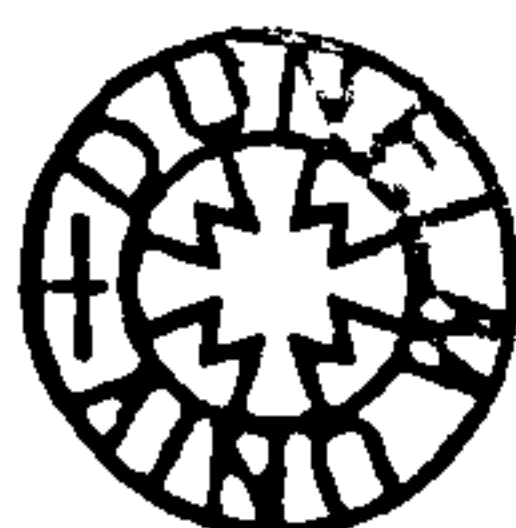
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*Submitted for Doctor of Philosophy in Economics*

**University of Durham**

**Department of Economics and Finance**

**2005**



**05 MAY 2006**

## **ABSTRACT**

### **Essays on Inflation Targeting, Price Stability and The Conduct of Monetary Policy Under Imperfect Credibility**

by Anamaria Nicolae

The main aim of this thesis is to study the impact of lack of credibility in a transition to price stability on both, the behaviour of the output and the optimal speed of disinflation. The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices. The effects of a disinflationary monetary policy are studied when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price-level. However, an expectation updating rule that incorporate a more flexible way of modelling the evolution of agent's priors than previously done in the literature has been employed.

Previous work by Ball (1994) and Ireland (1997) identifies the "disinflationary booms" which accompany a disinflation program. They explain their appearance as due to imperfect credibility. In this thesis it is demonstrated that the "disinflationary booms" may or may not disappear in an environment with imperfect credibility, depending on the speed of learning relative to the speed of disinflation. Regarding the optimal speed of disinflation, this thesis suggests that in both cases of perfect and imperfect credibility big inflations should be stopped rapidly. On the other hand, in the case of imperfect credibility, for small inflations, the speed of disinflation decreases in comparison to the case of perfect foresight. Also, once inflation has risen substantially, imperfect credibility makes sizeable output losses in the transition to price stability highly likely even when the speed of disinflation is 'optimal'.

The material contained in this thesis has not been previously submitted for a degree in this or any other university.

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Work from this thesis is forthcoming in Nicolae and Nolan, "The Impact of Imperfect Credibility in a Transition to Price Stability", *Journal of Money, Credit and Banking*.

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# Chapter 1

## Introduction

The purpose of this thesis is twofold. First, it aims to present how the problem of imperfect credibility (Kydland and Prescott, 1977 and Barro and Gordon, 1983) has been tackled by the implementation of inflation targeting. The motivation stems from the work of Woodford (1999), Gali (2000), Svensson (2000, 2003) which states that the success of the monetary policy depends on the credibility of monetary authority that pursues it. Second, it aims to extend the study of the conduct of monetary policy in a transition to price stability under conditions of imperfect credibility.

The first aim is addressed in the second chapter of the thesis. This chapter reviews the literature on monetary policy rules, inflation targeting and imperfect credibility. My literature review draws on the work of Taylor (1993, 1999, 2000), McCallum (1987, 1995, 1999), Ball (2000) and King (1999). All these papers have a common theme. They argue for the efficiency of policy rules in improving economic performance. However, no rule describes how policy should be generally set. Therefore, some discretion is inevitable and as King (1999) states, this discretion must be led or constrained by a clear objective. With this perspective I move on to study inflation targeting which is the central part of the literature review. The presentation of the main achievements of inflation targeting is followed by a description of the frameworks of inflation targeting including both the institutional and the operational frameworks.

The costs of a disinflation are generally believed to depend on the credibility of the central bank's commitment to the new lower inflation target<sup>1</sup>. Therefore, I present some of the important recent contributions regarding the conduct of a disinflationary monetary policy in a New Keynesian setting, where the supply-side of the economy is

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<sup>1</sup>Antulio and Rudebusch (2000).

characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices.

For the purpose of the second aim of this thesis, three broad results stand out from the reviewed literature concerning the effects of a monetary contraction in such an environment. First, in periods following a contraction in the money stock, real output is likely to fall below its long-run equilibrium level, now altered. Second, a gradual disinflation may actually result in output, after its initial decline, rising above its new steady-state level and remaining there for some time. Third, it is optimal to end high inflations quickly while low inflations are best ended gradually, and also to maintain inflation at or near zero thereafter. The key papers that develop these results are due to Ball (1994), Ireland (1997), King and Wolman (1999) and Khan, King and Wolman (2002).

The theoretical papers addressing these issues and many others besides, assume perfect foresight (or rational expectations). For some purposes this assumption is obviously appropriate, indeed, what other assumption makes sense when one wishes to analyse an economic model in, or in a close neighbourhood of, an unchanging steady state? However, the assumption of perfect foresight may be less attractive when one wishes to analyse the effects of 'large' changes in policy. The steady state of the model may well be changing and in addition, policymakers may not enjoy complete credibility. This thesis extends the examination of the effects of a disinflationary monetary policy when policymakers initially do not enjoy complete credibility and where the steady state of the economy is changing.

The idea of introducing imperfect credibility into a New Keynesian settings is not new. For example, in Ball's (1995) model, agents harbour a suspicion that the authorities will renege and give up on the path of disinflation. He models agents scepticism as a constant conditional probability of renegeing. This may be a rigid way of modelling the evolution of agents' priors. This is due to the fact that as the disinflation proceeds, agents may accord increasing weight to the announced path for the money supply. However, as the disinflation proceeds, the extent of nominal rigidity in the economy also optimally rises. Therefore, the authorities may be more likely to renege (to exploit the Phillips curve). My thesis argues that both these cases are intuitively plausible. Therefore, I propose an 'expectation updating rule' that nests both these

alternatives. I model this time improving credibility by introducing a way to capture the fact that the probability mass which characterizes the agent's subjective expectations is evolving over time towards the central's bank announced path of the money supply. Moreover, in this thesis, this movement can take place in three different ways, each of them capturing different paths of the evolution of private sector willingness to believe the government's announcement about the money supply after the change in policy. This evolution is seen as some form of learning.

My thesis also draws on Ireland (1995) who also finds that higher output losses are the price of imperfect credibility during a period of disinflation. Again, like Ball, his modelling of expectations formation is not flexible enough, as it fails to incorporate time improving credibility which is my principal contribution. To reiterate, my thesis bridges a gap in the literature by implementing a more flexible way of modelling imperfect credibility in a New Keynesian model. It then examines the behaviour of aggregate output and the optimal speed of disinflation in this new setting.

The outline of the thesis is as follows. Chapter 3 presents the model employed to explore the above mentioned issues and also discusses its salient features. This model allows us to conduct the analysis in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices. It allows us to study the effect of a monetary policy that brings money growth to zero over some horizon. This was the approach adopted by Ireland (1997), following Ball (1994).

Chapter 4 discusses the case of imperfect credibility and presents the way this has been modelled. The implementation of imperfect credibility leads to some computational complexities related to the optimal choice of prices by firms who not only have to forecast future demand and cost conditions, but also have to forecast their covariances. Chapter 5 presents the methodology for solving this new model.

Chapter 6 presents the output effects of a period of disinflation while there is a temporary lack of credibility. Two important recent contributions address some of the issues also discussed here. Ball (1995) demonstrates that if credibility is sufficiently low, a period of disinflation may lead to expected output losses and Ireland (1995) finds that higher output losses reflect the cost of imperfect credibility during a period of disinflation. However, the attainment of price stability is desirable (i.e., welfare

enhancing) in general, except when the loss of seigniorage is replaced in the low inflation state by a rise in other distortionary taxes. As presented earlier, their modelling of the expectations formation process misses the effects that I wanted to analyse.

Two major results emerge from my thesis. First, the impact of imperfect credibility is to make disinflation more costly in terms of output losses in the period immediately following the contraction in monetary growth. The path of output to its new steady state differs from the path under perfect foresight. Second, the main insight arising from this work is that the ‘disinflationary booms’ found by Ball (1994) and King and Wolman (1999) may or may not disappear in an environment with imperfect credibility, depending on the speed of learning relative to the speed of disinflation.

Chapter 7 focuses on the extension of Ireland’s (1997) calculation of the optimal speed of disinflation to the case of imperfect credibility, and enquires whether or not imperfect credibility materially impacts on the optimal speed of disinflation, as compared with the situation under perfect foresight. This is a question of first-order policy importance which has not been addressed hitherto in the class of models employed in this thesis. This model is also used to analyse the Volcker disinflation and discusses whether the speed of the Volcker disinflation was excessive or not. Given the history of disinflation I infer the implied speed of learning using my model economy. Then, using this implied speed of learning, the speed of the Volcker disinflation is compared with the optimal speed of disinflation.

The policy lessons that emerge from this exercise is that big inflations should be stopped quickly, in the instances of both perfect and imperfect credibility. However in the case of imperfect credibility, the speed of disinflation of small inflations is decreased in comparison to the case of perfect foresight. It also suggests that once inflation has risen substantially, imperfect credibility makes sizeable output losses in the transition to price stability highly likely even when the speed of disinflation is ‘optimal’.

Chapter 8 concludes.

# Chapter 2

## Literature Review

### 2.1 Introduction

Imperfect credibility has recently become the focus of much research. The credibility problem was first introduced in macroeconomic policy by Kydland and Prescott (1977) and Barro and Gordon (1983). Since then, many papers have contributed to develop this subject which has increasingly taken a central role in the conduct of monetary policy.

The impact of imperfect credibility on the effectiveness of monetary policy was, in most cases, studied in the context of the time inconsistency problem of Kydland and Prescott (1977). In their paper, they showed that the monetary authority has an incentive to use discretionary policy to generate price surprises when there is a negative relationship between unemployment and unanticipated inflation. The rational agents recognize the potential for real wage reductions and bid up wages, thus causing inflation to increase without any compensating decrease in unemployment. Hence, discretionary monetary policy leads to inflationary bias in economies.

The success of monetary policy depends on the credibility of the monetary authority that pursues it (Woodford (1999), Gali (2000), Svensson (2000, 2003)). Credibility arises from constant communication with the public, from commitment and consistency. As Blinder (1999) emphasizes, there is no unanimously agreed-upon definition of credibility in the literature. Blinder's favourite definition, is that deeds are expected to match words: "A central bank is credible if people believe it will do what it says"<sup>1</sup>. However, nowadays, "a central bank's or policymaker's 'credibility' is the extent to

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<sup>1</sup>Blinder (2000), p: 1422.



which the private sector believe there will be low inflation”<sup>2</sup>.

Clearly, the implementation of monetary policy rules is an attempt to tackle the problem of imperfect credibility by providing a committed and consistent way of conducting monetary policy. As argued by Taylor (1993), policy rules are superior to discretion in improving economic performance as they help to improve credibility. The literature reviewed for the purpose of this thesis presents in the first instance a discussion of monetary policy rules. The section dedicated to monetary policy rules will present their importance, the historical context of their adoption followed by the general framework used for the research of monetary policy rules. The development of monetary policy rules over time and the current areas of controversy and debate will also be discussed.

Moreover, time inconsistency appears no longer to be a problem under the inflation targeting regime (Bernanke et al. (1999)). Inflation targeting is characterized by the announcement of an official target (or range) for the rate of inflation for the coming period, a commitment to price stability as the primary goal of monetary policy and an information inclusive strategy (the central bank makes use of all available information). Therefore, the focus of the next section of the literature review is on inflation targeting. Inflation targeting increases transparency through communication to the public and the financial markets of the monetary policy decisions and of the targets. Also, it provides for increased accountability of the central bank for meeting these objectives (Mishkin, 2000). This is a monetary policy regime designed to address and correct the problem of imperfect credibility. After a discussion of the achievements of inflation targeting, a description of the framework of inflation targeting including both the institutional and the operational framework will be presented. The section will end with a discussion of the adoption of inflation targeting in transition economies.

However, as Antulio and Rudebusch (2000) states, “The costs of a disinflation are also commonly believed to depend on the credibility of the central bank’s commitment to the new lower inflation target”<sup>3</sup>. Therefore, this leads to another aspect in respect to which imperfect credibility will be discussed in this chapter. More specifically, there

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<sup>2</sup>Forder (2004), p: 416.

<sup>3</sup>Antulio and Rudebusch (2000), p: 708.

will be a discussion of the behaviour of output during a disinflationary monetary policy and of the optimal speed of disinflation when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price-level.

The outline of the chapter is as follows: Section 2.2 presents the importance and the evolution of the monetary policy rules, followed by a review of the inflation targeting regime in Section 2.3. It reviews the historical roots, describes the framework of inflation targeting and briefly discusses the adoption of inflation targeting in transition economies. The conduct of disinflationary monetary policy under imperfect credibility and the optimal speed of disinflation are discussed in section 2.4. Section 2.5 concludes.

## 2.2 Monetary Policy Rules

The credibility of the central bank is essential in the pursuit of a successful monetary policy (Svensson 2000, 2003). Credibility is based on constant communication with the public, *commitment* and *consistency*. The implementation of monetary policy rules aims to address the problem of imperfect credibility as it provides both a committed and consistent manner of conducting monetary policy. Central bankers emphasised the importance of following a more systematic policy, especially with credible, clearly stated goals, specifying the monetary policy rules as a guide for decisions. The importance of monetary policy rules is directly related to the rules versus discretion policy debates initiated by Kydland and Prescott (1977), Barro and Gordon (1983) and Taylor (1993) who argue for the superiority of policy rules over discretion in improving economic performances.

This section first presents the importance of monetary policy rules, followed by the historical context of the adoption of monetary policy rules in practice and the general framework used for the research of monetary policy rules. It also presents the developments of the monetary policy rules over time and the current areas of controversy and debate, since monetary policy rules have become a well-known subject of research.

Lucas's "Econometric Policy Evaluation Critique", clearly indicates that in order to evaluate a monetary policy, there is a need for rules: people are forward-looking and a policy is described by a rule. A policy rule, representing a description of how the instrument of policy changes in response to economic variables, is a guideline for

monetary policy. It is also defined as a contingency plan that lasts forever only if there is an explicit clause not to abandon it. To have meaning, a policy rule has to be in place for a reasonably long period. In order to gain the credibility associated with a rule, policymakers need to make a commitment to the rule (Taylor (1993)).

Advocating *the importance of monetary policy rules*, Taylor (1999) enhanced the effects of the monetary policy rules on macroeconomic stability, stating that more responsive policy rules lead to a greater economic stability, although this evidence is not so simple due to various shocks. Longworth and O'Reilly (2000) also pointed out their importance, arguing that a stable reaction function helps in the analysis of shocks and of the implications new information can bring. The monetary rule is therefore an important part of the model used for predicting the future evolution of macroeconomic variables.

According to Taylor (1998), the advantages the adoption of policy rules brings are:

- Time consistency. A policy rule reduces the chance that the policymakers change their policy after the private sector has taken its action. Kydland and Prescott (1977) highlighted this rationale for policy rules.
- Clearer explanation. A better educated public and more effective democracy.
- Less short run policy pressure. A monetary policy rule that shows how the instrument of policy has to be set in a large number of circumstances is less subject to political pressure in times of change.
- Reduction in uncertainty by clearly describing future policy actions.
- Teaching the art and science of central banking.
- Greater accountability.
- A useful historical benchmark to provide a baseline for historical comparison.

Although subject of research long before adoption, monetary policy rules were applied in practice quite recently. It is a little strange to speak about the *historical context* of the adoption of monetary policy rules in practice when it only began two decades ago. However, let this be considered as a reference point in the understanding of the evolution of the monetary policy rules over time.

In 1980's, in the US, monetary policy was characterized by discretion with many commentators describing monetary policy as following the "Greenspan standard". This refers to the way in which Alan Greenspan, the Federal Reserve Chairman, was leading the short term interest rate in a purely discretionary fashion. Therefore, there was a gap between research and practice. Chapter 3 of the *1990 Economic Report of the President* was the first attempt to translate academic research on policy rules into a potential tool that could perhaps be used in practice by policymakers. This report helped to smooth the way for the application of policy rules in practice. In 1991 Taylor finalized a book on using econometric rational expectations based models to evaluate monetary policy rules, a project started in the mid 1980's, as an extension of a 1970's research agenda.

A seminal work in this field is Bryant, Hooper and Maan (1993) that summarizes many rational expectations models in a comparison project of monetary policy rules. Although there was some disagreement among economists about which monetary policy rule is most appropriate for adoption, a consensus emerged from the results. Taylor (1998) summarizes this consensus thus:

1. An interest rate instrument performs better than a money supply instrument.
2. Interest rate rules that react to both inflation and real output worked better than rules focused on either one.
3. Interest rate rules reacting to the exchange rate were inferior to those that did not.

As a result, at the Carnegie Rochester Conference on Public Policy in Pittsburgh in November 1992, at the invitation of Alan Meltzer, Taylor presented the first monetary policy rule, now called the *Taylor rule*. Presented later in this section, it fulfils the three characteristics stated above.

One aspect needs to be addressed at this point and this is the type of model employed for the research of monetary policy rules. The framework usually met consists of a structural quantitative macroeconomic model that includes three main components (McCallum, 1999):

- An IS relation or set of relations specifying how the interest rate affects aggregate demand and output.

- A price adjustment equation or set of equations relating inflation's behaviour to the output gap and to expectations regarding future inflation, i.e. New-Keynesian Phillips curve.
- A monetary policy rule specifying each period's settings of an interest-rate instrument.

These settings are determined considering the most recent or predicted values of the inflation rate and the output gap for the economy. Estimated by various methods, or perhaps calibrated, the models usually used are quarterly and most of them incorporate rational expectations. However, according to Taylor (1998), either calibrated or estimated, the models are based on five principles that have guided research on monetary policy rules:

1. Long-term trend of real GDP is well-described by modern neoclassical growth theory, having the growth rate of productivity depending on capital and technology, both endogenous and susceptible to changes in tax policy, trade policy and regulatory policy. The trend in GDP is not constant, but it changes slowly.
2. There is no long-run trade-off between inflation and unemployment (or deviations between real GDP and the long-term trend in real GDP).
3. There is a short-run trade-off between inflation and unemployment (due to sticky prices or imperfect information). Changes in monetary policy have a short-run impact on unemployment even though monetary policy is neutral in the long run.
4. Although there are still debates about how to model expectation, rational expectation has become the baseline assumption; expectations for the future count in the evaluation of monetary policy, and are endogenous to changes in policy.
5. Monetary policy needs a target inflation rate with a low inflation target being preferred to a high one.

Consistent with the second principle, there is no need to establish a target for unemployment (monetary policy has no effect on unemployment in the long run). Due to the shocks that will shift the economy from its target, a set of procedures is needed to change the instruments of policy in response to these shocks. According to Taylor

(1993), a procedure can be described by a policy rule. The third principle enhances the importance of the policy rule for the instrument due to the fluctuations of inflation and unemployment that affect each other in the short-run. That expectations matter and are endogenous was enough of a reason for the models to be focused on the evaluation of policy rules.

Furthermore, since monetary policy rules have been the subject of much recent research, a survey of how monetary policy rules developed over time will be presented. A seminal paper in this direction is Taylor (1999) who presents a collection of articles which by applying various econometric frameworks, investigate the robustness of alternative monetary policy rules. Also, McCallum (1999) offers a review of some of the recent developments in the empirical analysis of monetary policy rules.

The history of changes in monetary policy rules is relevant for monetary policy today because it provides evidence about the effectiveness of different monetary policy rules. To study the effects of policy rules behaviour, stochastic simulations using the models with alternative policy rules are conducted. The results are ranked according to the performance in terms of average values of the variability of inflation, of the output gap and the interest rate. One monetary policy rule is better than another monetary policy rule if it results in better economic performance according to some criteria such as lower variability of output and inflation.

Taylor (1998) discusses an historical approach to the evolution of monetary policy rules arguing that this may be more convincing for policymakers than the model-based approach due to the fact that first, models are viewed (even simple ones) like black boxes and second, because the evolution of monetary policy rules over time is a real-time learning process. The evolution of monetary policy rules is best understood as a gradual process of learning how to conduct monetary policy. He centres his analysis on the short term interest rate instruments of the central bank, while examining several long periods in US monetary history: the classical international gold era, the fixed exchange rate era of Bretton Woods and flexible exchange rate era. Taylor (1999) points out that macroeconomic performance was changed with different policy rules over each of these three periods (in terms of volatility of inflation and output). He surveyed the effects of the (different) policy rules on macroeconomic stability, stating that the more responsive policy rules lead to greater economic stability, although this

evidence is not so simple due to various shocks. The degree of responsiveness of short-term interest rates in the monetary policy rule to fluctuations in inflation and output increases over time, from the classical international gold era, the fixed exchange rate era and the flexible exchange rate era together with the economic stability that characterize each of them. The importance of having a rule to guide policy became apparent when the Bretton Woods's system fell apart in the early 1970's.

There are different types of monetary policy rules. In the following section, the main classes of policy rules, starting with the Taylor rule and its extensions will be presented.

### 2.2.1 Taylor Rule

Introduced by Taylor (1993), the Taylor rule was derived for the US economy, showing that the monetary policy between 1986-1992 is well characterized by a rule for the Federal Reserve's interest rate instrument. Moreover, monetary policy since 1992 has also been well described by this policy rule, and not only for the Federal Reserve but quite accurately<sup>4</sup> also for the Bundesbank and other central banks . He finds that the nominal interest rate (the nominal Federal funds rate), the Federal Reserve's instrument, responds with fixed, positive weights to inflation and the output gap. Moreover, the Taylor rule for the Federal funds rate fits the actual policy performance during the last few years remarkably well.

The Taylor rule, as presented in McCallum (1999, 2000):

$$R_t = \Delta p_t^a - 0.5(\Delta p_t^a - \pi^*) + 0.5\tilde{y}_t + \bar{r},$$

where:

$R_t$  is the short-term nominal interest rate that the central bank uses as its instrument,

$\Delta p_t^a$  is the average inflation rate over the past four quarters, a proxy for expected inflation,

$\pi^*$  is the inflation target,

$\tilde{y}_t = y_t - \bar{y}_t$  is the output gap (percentage difference between the actual and capacity output values),

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<sup>4</sup>see Clarida, Gali and Gertler (1997).

$\bar{r}$  is the average real rate of interest.

In Taylor (1993), it is presented as follows:

$$r = p + 0.5y + 0.5(p - 2) + 2,$$

where:

$r$  is the federal funds rate,

$p$  is the rate of inflation over the previous quarters,

$y$  is the percent deviation of real GDP from target, given that he chose for the average real rate of interest 2% and for the inflation target he assumed 2%.

The Taylor rule is a linear reaction function that responds in deviations of inflation from its target and to the output gap, according to which monetary policy should be tightened by an increase in interest rate when inflation exceeds its target value and/or output exceeds capacity. However, there are developments to this rule that will be presented over the next paragraphs. For a better understanding of the evolution of the extensions of the Taylor rules, the rule below will be written in a simplified form, as follows:

$$i_t = i_t^* + q_\pi(\pi_t - \pi^*) + q_y(y_t - y^*), \quad (2.1)$$

where:

$i_t$  is short term nominal interest rate that the central bank uses as its instrument,

$i_t^*$  is “neutral” nominal interest rate,

$\pi_t$  is the inflation rate,

$\pi^*$  is central bank’s target inflation rate,

$y_t$  is logarithm of the level of actual output,

$y^*$  is logarithm of the trend output,

$q_\pi, q_y$  are positive parameters corresponding to how active monetary policy is in responding to deviations of output from trend (output gap) and inflation from its target. (It can also specify the setting of a nominal interest rate instrument in response to observed or predicted values of inflation and the output gap for forward-looking Taylor rules which will be presented later).

#### *Evolution of the parameters of the Taylor rules over time*

The initial parameters of the original Taylor rule were  $q_\pi = 1.5$  and  $q_y = 0.5$ . Over



time, the parameters of the policy rule changed, their magnitude depending on how monetary policy is run, on how active monetary policy is in responding to deviations of output from trend and inflation from its target. The size of the coefficients makes a big difference for the effects of policy. Nelson (2000) reached the same conclusion as Taylor (1999) regarding the relationship between the size of parameters and the degree of responsiveness of the rule, testing the Taylor rule for UK for 1971-97 in different eras. It is concluded that the degree of responsiveness of short-term interest rates in the monetary policy rule to fluctuations in inflation and output increases with the size of the parameters (parameters greater than one characterize periods of low inflation), as well as the economic stability. Taylor's (1999) study on the evolution of the Taylor rule coefficients over time from the classical international gold era, to the fixed exchange rate era and the flexible exchange rate era, while testing it for the US economy, provides empirical evidence in this point.

## 2.2.2 McCallum Rule

Before investigating further the extensions to the Taylor rule, I turn next to present the McCallum rule. Some of the studies consider alternative instruments or target variables (McCallum and Nelson, 1999). For example, McCallum (1987, 1993, 2000) promoted a rule that features a monetary base instrument and a nominal income target. The McCallum rule is:

$$\Delta b_t = \Delta x^* - \Delta v_t^a + 0.5(\Delta x^* - \Delta x_{t-1}),$$

where:

$\Delta b_t$  is the change in the log of the adjusted monetary base,

$\Delta x^*$  is a target growth rate for nominal GDP,

$\Delta x_t$  is the change in the log of nominal GDP,

$\Delta x^*$  is specified as  $\pi^* + \Delta y^*$ , where  $\Delta y^*$  is the long run average rate of growth of real GDP,

$\Delta v_t^a$  is the average growth of base velocity over the previous 16 quarters,

$v_t = x_t - b_t$ , is the log of base velocity, intended to reflect long lasting changes in the demand for the monetary base.

This rule adjusts the base growth rate up or down when nominal GDP growth is below or above a chosen target value,  $\Delta x^*$  which equals the desired average rate of

inflation plus the expected long-run average rate of growth of real output.

When comparing the Taylor rule and the McCallum rule, McCallum (1999) stated that the Taylor rule is much more popular because it is expressed in terms of the interest rate rather than in settings for the growth rate of the adjusted monetary base (currency plus bank reserves). At the same time, it is more realistic in the sense of being more appropriate for the central bank's actual instrument variable. Many central banks conduct their policy in terms of the interest rate rather than the monetary base. This does not mean that the McCallum rule does not offer consistent advice or that it should not be considered by policymakers concerning monetary policy. Moreover, according to McCallum (1999) one of the weaknesses of the Taylor rule is that it requires estimating the output gap which is difficult whereas this is not needed for the McCallum rule.

Empirically, these two rules have agreed over many periods. They were different in the case of the UK over the late 1980s when the Taylor rule was pointing to an easing in policy, while McCallum argued for a tighter one. The empirical study of McCallum (1993) in the case of Japan for 1972-1998 provided evidence that the policy rule that used the monetary base as a variable would identify that the policy was too tight after 1991. He argues that this would have delivered a better monetary policy.

Although there is no similarity, regarding the instrument and the target variable, both the Taylor rule and the McCallum rule serve the same goal of a monetary policy rule designed to keep inflation low and stable and to stabilize real output fluctuations. However, there remains an extremely important role to be played by measures of the monetary base and other monetary aggregates.

Given the importance of these two types of rules, their advantages and disadvantages, McCallum (2000) studies rules obtained by pairing interest rate instrument and monetary base growth instruments with both a nominal income growth target and inflation as target variables. The rules considered are:

$$R_t = \bar{r} + \Delta p_t^a + 0.5(\Delta x^* - \Delta x_{t-1})$$

and

$$\Delta b_t = \Delta x^* - \Delta v_t^a - 0.5h_t,$$

where  $\bar{r}$  is the long run average real rate of interest,  $h_t = \Delta p_t^a - \pi^* + \tilde{y}_t$  is the "hybrid"

target value. Also

$$R_t = \bar{r} + \Delta p_t^a + 0.5(\Delta p_t^a - \pi^*)$$

and

$$\Delta b_t = \Delta x^* - \Delta v_t^a - 0.5(\Delta p_t^a - \pi^*).$$

His empirical study for the case of the US, the United Kingdom and Japan investigated the performance of these rules. He provided evidence that, by combining the interest rate and the monetary base instrument with both inflation and nominal income as target variables, the performance of policy rules seems to be more dependent on the instrument variable than on the choice of variable target. This is an important topic for future research concerned with alternative monetary policy rules.

### 2.2.3 Extensions of Taylor Rule

There is a large body of literature based and focused on extension of the famous Taylor type rule. The last decade has been characterized as a period of innovations in monetary policy, when different countries have experimented with different techniques for achieving inflation targeting, including different choices of policy rules and policy instruments.

A common and major change discussed by Ball (2000) and King (1999) has been to include the instrument lagged on the right hand side as a determinant of the contemporaneous value of the instrument:

$$i_t = i^* + q_\pi(\pi_t - \pi^*) + q_y(y_t - y^*) + q_i i_{t-1}, \quad (2.2)$$

where  $q_i$  is a positive coefficient.

This adjustment is intended to reflect the practice of interest rate smoothing with the interest rate depending also on its previous level. It captures the aversion of the policymakers to a large shift in the instrument, in favour of a cautious, gradual approach, as discussed by Ball (2000). In fact this is a more “aggressive” response to shocks to inflation or output than the previous rule (2.1) (King (1999)). Apparently, a prompt response to shocks may present the need for larger successive movements, hence a less volatile path for the interest rate. The relation between these two rules, (2.1) and (2.2), depends on the assumption about the nature of the transmission mechanism

of monetary policy.

In order to present estimates for the UK economy for 1972-97 in his attempt to characterize the monetary policy using Taylor rules with smoothing, Nelson (2000) employed a Taylor rule in which output appears with a one period lag, rather than contemporaneously:

$$R_t = w_0 + w_1 \Delta p_{t-1} + w_2 \tilde{y}_{t-1},$$

where  $w_0, w_1, w_2$  are parameters of the rule. This reflects a more realistic assumption about the information available to the monetary authority in period  $t$ , incorporating Orphanides (1999) suggestion that there are differences between real-time and final data, which usually affects the analysis of policy rules. This is one of the reasons Orphanides (1999) criticised the Taylor rule. He recognised that current-period values for  $y_t$  could not be known until after the end  $t$ , and also that the bulk of problems is due to the errors in the measurement of potential output.

#### 2.2.4 Open-Economy Rules

As stated by Svensson (1998), in the Taylor rule the instrument is restricted to respond only to the deviations between the variables and their target level, possibly including the lagged instrument in order to incorporate instrument smoothing. In its standard form, the Taylor rule is a backward-looking rule for a closed economy. This rule was derived for the closed economy of the US. Taylor (2000) explained the formulation of a modified rule by the fact that the central bank reacted too strongly to the exchange rate and that inflation-output performance and that the monetary policy rule should concentrate on domestic macroeconomic variables which was not allowed for in the earlier rule.

As Ball (2000) stated, monetary policy in open economies differs from policy in closed economies. In the first ones, macroeconomic stability is enhanced by targeting a measure of inflation that filters out the transitory effects of exchange-rate fluctuations. Ball (1999a) proposed a Taylor rule with the exchange rate included, as a rule designed for a small open economy, arguing that policy rules developed for closed economies are inadequate for responding to shocks to the exchange rate, exports and world commodity prices. Therefore, for open economies, the Taylor rule must be modified so as to give a role to the exchange rate. Moreover, as Debelle (1999) pointed out, by including the

exchange rate, the central bank can reduce the variability of both output and inflation.

Thus, adding the exchange rate, the rule becomes:

$$i_t = r^* + q_\pi(\pi_t - \pi^*) + q_y(y_t - y^*) + q_e e_t, \quad (2.3)$$

where  $e_t$  is the log of the real exchange rate (a higher  $e$  means appreciation) and  $q_e$  is a positive coefficient. A lagged exchange rate has been included as well:

$$i_t = r^* + q_\pi(\pi_t - \pi^*) + q_y(y_t - y^*) + q_{e0}e_t + q_{e1}e_{t-1}.$$

As shown by Ball (1999b), using a very simple model with constant coefficients (Svensson-Ball model), in an open economy, the optimal rule changes in two ways. First, the policy instrument becomes a weighted average of the interest rate and the exchange rate, what is often called an MCI<sup>5</sup>. As it can be seen, here is a change in the instrument, while most of the central banks use a short-term interest rate like in the above rules. Knowing that inflation targeting in open economies may lead to high variability of exchange rate and output, these fluctuations can be avoided by targeting long run inflation. Therefore, the second change consists of the replacement of inflation by long run inflation which is a variable that filters out direct but transitory effects of exchange rate-movements. Thus, the optimal policy rule in a small open economy, it is argued, is similar to the Taylor rule, differing from the original by not being expressed in terms of the interest rate but in terms of an index and targeting long run inflation.

Svensson (1998) comments on the role of MCIs within inflation targeting. He concludes that the monetary policy impact on inflation that is transmitted via several different channels with different lags is too complex to be summarized by any single index. In an open economy, an instrument that combines both (the main channels of transmission, i.e. exchange rates and interest rate) would be desirable due to the fact that it summarizes the stance of monetary policy offering a better instrument.

Canada, New Zealand, Sweden (which follows the Canadian monetary policy closely) and Norway, pursue this approach. The rationale for using a MCI is that it is a measure of the overall stance of policy, including the incentives through both the real interest rate and the exchange rate. Recently it has been studied for the UK. Until now, New

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<sup>5</sup>Monetary Condition Index.

Zealand and Canada adopted an MCI as an instrument, this still being a relatively unexplored area that might raise questions for the future research.

According to Ball (1998), the optimal policy as a rule for an average of  $r$  and  $e$  is:

$$wr_t + (1 - w)e_t = a\tilde{y}_t + b(\pi_t + \gamma e_{t-1}), \quad (2.4)$$

where  $r_t$  is the real interest rate,  $\tilde{y}_t$  is log of real output,  $a$ ,  $b$  positive constants to be determined, and  $\gamma$  positive coefficient,  $w$  is the weight on the real interest rate ( $0 < w < 1$ ) and  $wr_t + (1 - w)e_t$  is the monetary conditions index (MCI).

A new element is  $\gamma e_{t-1}$  interpreted as a long-run forecast of inflation under the assumption that output is kept at its natural level. While in a closed economy this will be simply the current inflation, in an open economy inflation will change due to the exchange rate.

Ball (2000) pointed out that theoretically, equations (2.3) and (2.4) are identical, differing in the choice of the instrument between an interest rate and an MCI as policy instrument. This choice depends on the degree of flexibility allowed for the policymakers. A question that requires more research is whether the exchange rate should appear on the left or right side of the rule, in other words, if the policy instrument should be an interest rate or an MCI.

Recently, as presented by Longworth and O'Reilly (2000), researchers at the Bank of Canada, noticed that Ball's results depend on assumptions that are not true in the case of a small open economy (shocks are white noise and in particular are uncorrelated across equations). Therefore, they extended the rule to include new exogenous explanatory variables  $X$ , due to the fact that commodity prices and foreign output affects both the exchange rate and demand.

The optimal policy rule becomes:

$$wr_t + (1 - w)e_t = ay_t + b(\pi_t + fe_t - 1) + cX_t,$$

where all variables are measured as deviations from their average values,  $wr_t + (1 - w)e_t$  represents the monetary conditions index (MCI),  $y_t$  is the output gap,  $(\pi + fe_t - 1)$  is core inflation, a measure of inflation that excludes the direct but temporary effects of exchange rate movements (concerning the measurement of inflation there is an on-going debate within the inflation targeting literature);  $c$  is a constant vector that depends

on the parameters of the model excluding the coefficients on  $X_t$  in the exchange rate equation. The instrument responds to changes in  $X$ , since this affects the future path of output and inflation. Srouf (1999) noticed that an autonomous rise in the exchange rate requires constant monetary conditions, while a rise due to an increase in real commodity prices require tighter monetary conditions.

It would be desirable to a central bank to have a Taylor type reaction function incorporating Ball (1999a) and Srouf (1999) extensions for the internal discussion on monetary policy, robust across several models. While more work needs to be done to refine and understand the results of research in this area, the most general observation is that the results from the open-economy Taylor rule show less cycling for inflation, interest rates, the exchange rate and the output gap than do the results for the other two forms of the Taylor rule considered. These results should not be all that surprising given the more complete representation, relative to the variants of the base Taylor rule used, of how policy is actually formulated and implemented. Further research will continue on refining the conceptual underpinnings of an open economy Taylor rule and at the same time a concrete effort will be made, using simulation analysis and different models, to find a robust representation of a Taylor rule that can be used to aid in the discussions on monetary policy formulation.

### 2.2.5 Forward-Looking Rules

We now consider forward-looking approaches to policy rules. Due to the fact that inflation reacts with “long and variable lags” and with variable magnitude to changes in the monetary policy instrument and that it is also affected by factors other than monetary policy (sometimes with shorter lags), the central bank has to adopt a forward-looking perspective by attempting to control inflation a few years ahead (Svensson (2000)). The degree of forward-lookingness is usually thought to be determined by the transmission lag. This forward-looking perspective has long been a consideration. Indeed, Keynes observed that “if we wait until a price movement is actually afoot before applying remedial measures, we may be too late”<sup>6</sup> [*A Tract on Monetary Reform* (1923)].

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<sup>6</sup>Keynes (1923), p: 148.

## Forward-looking Taylor Rules

In its standard form, the Taylor rule is a backward-looking rule, while inflation targeting is a forward-looking regime given the lags in the effect of monetary policy on the real economy and inflation. However, a policy rule including explicitly forecast values of output and inflation becomes a forward-looking policy rule. Consequently, due to the fact that it takes direct account of the policy lags it delivers superior outcomes. Moreover, Debelle (1999) provided evidence that such a rule reduces the variability of both inflation and output. Therefore, the famous Taylor rule uses estimated (forecasted) values of inflation rather than current inflation. Such a rule is present in McCallum (2000):

$$i_t = i^* + q_\pi(E_t\pi_{t+1} - \pi^*) + q_y(y_t - y^*).$$

Nelson (2000) in his empirical study on the case of UK (July 1976 to April 1979) testing various rules, used a rule involving an estimated contemporaneous value for the output gap as well:

$$i_t = i^* + q_\pi(E_{t-1}\pi_t - \pi^*) + q_y E_{t-1}\bar{y} + q_i i_{t-1}. \quad (2.5)$$

Advocating the forward-looking nature of inflation targeting, Haldane (1997) presents three forward-looking rules and states that the monetary authority in inflation targeting countries UK, Canada, New Zealand, feeds back from expected rather than actual inflation. The rules he considers are as follows:

$$i_t = q_\pi(E_t\pi_{t+j} - \pi^*), \quad (2.6)$$

$$i_t = i^*, \quad (2.7)$$

policy feeding back from current inflation, and

$$i_t = q_\pi(\pi_t - \pi^*), \quad (2.8)$$

called “myopic rule”, where:

$E_t$  is the expectations operator conditional on information at time  $t$  and earlier,  
 $q_\pi$  is a positive feedback parameter,



$j$  is the targeting horizon (determined among other things by the length of the monetary transmission channel).

The empirical study of Clark, Laxton and Rose (1995) on rules (2.6) and (2.8) provided evidence that myopic rule results in greater output variability than rule (2.7), pointing out that the use of an estimated/forecasted value instead of the current one led to a better result. Furthermore, as claimed by Haldane (1997), the effect that occurs among inflation targeting countries is *inflation forecast targeting*, with the *forecast* taking the role of the feedback variable, the lags embedded in the monetary transmission process being explicitly recognized in the setting of the monetary policy. In fact, as Alan Greenspan stated, “Implicit in any monetary policy action or inaction is an expectation of how the future will unfold, that is, a forecast”<sup>7</sup>.

### 2.2.6 Inflation Forecast Based Rules

The reaction function which involves adjusting interest rates to the expected future deviations of inflation from its target level, belongs to the class of inflation forecast based (IFB) policy rules. These rules allow for interest rate smoothing or include other terms (contemporaneous output gap (Canada)), while their main feature is forward-lookingness with respect to inflation. These kinds of rules are currently used in QPM<sup>8</sup> by the Central Bank of Canada. The class of rules is:

$$r_t = \gamma r_{t-1} + (1 - \gamma)r_t^* + \theta(E_t\pi_{t+j} - \pi^*),$$

where:

$r_t = i_t - E_t\pi_{t+1}$  is the short term *ex ante* real rate of interest,

$r_t^*$  is the equilibrium value of the real interest rate,

$E_t(.) = E(.|\phi_t)$ , where  $\phi_t$  is the information set available at time  $t$ ,

$E$  is the mathematical expectations operator,

$\pi_t = p_t^c - p_{t-1}^c$ ,  $p_t^c$  is the log of the consumer price index,

$j, \gamma, \theta$  parameters.

IFB rules have been examined by Batini and Haldane (1999) for the UK. They pointed out that such a rule is more efficient at minimizing inflation and output vari-

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<sup>7</sup> Alan Greenspan, quoted in Sir Alan Budd, “Economic Policy, with and without Forecasts,” *Bank of England, Quarterly Bulletin*, November, 1998.

<sup>8</sup> Quarterly Projection Model.

ability than standard Taylor rule specifications, and is almost as efficient as fully optimal rules (Srouf (2003)). Due to its simplicity this class of rules is more robust and easier to monitor than other rules.

### **2.2.7 Comparing forward-looking rules with closed economy Taylor rule**

Longworth and O'Reilly (2000) suggested that Taylor-type rules do not respond as well as forward-looking rules in terms of average stability. In their study, they compared the results from IFB-type rules, non-linear rules which do not react until inflation falls outside the bands, and Taylor-type rules that respond both to deviations of inflation from its target level and to deviations of output from potential using only contemporaneous or past information. On the basis of stochastic simulations with QPM, the results of the experiment were summarized in terms of output, inflation and interest rate variability. Nevertheless, in order to obtain more information for the monetary authority, researchers added an output gap term to the forward-looking rule, obtaining an extension of it, which makes it much closer to a Taylor type rule.

These rules are significant to the study of monetary policy because they confirm the results of Ball (1997) and Svensson (1997) as to the extent to which policy-making cares about output: the greater the weight on output the more gradual the response to deviation of the inflation forecast from the target value, which corresponds to a longer horizon of the inflation forecast in the policy rule.

Clearly, the implementation of monetary policy rules addresses and tackles the problem of imperfect credibility by reducing uncertainty by describing future policy actions, offering accountability and avoiding the problem of time inconsistency. They provide a consistent way of conducting the monetary policy and at the same time require commitment. After the first part of the literature review of this thesis dedicated to monetary policy rules, in which their importance, the historical context of their adoption and the general framework used for their research have been presented, attention is now turned to inflation targeting.

## 2.3 Inflation Targeting

The most popular monetary policy regime nowadays is inflation targeting due to the fact that it seems to provide a suitable framework to achieve economic stability, as shown by the experience of the countries that adopted this monetary policy regime. Inflation targeting is adopted at present by the following developed countries: Canada, the United Kingdom, New Zealand, Sweden, Australia, Finland, Spain and developing countries like Israel, Chile, Mexico and Brazil. Among the Central European Countries, the Czech Republic, Poland and Hungary have recently adopted inflation targeting.

“Inflation targeting is a framework for monetary policy characterized by the public announcement of official quantitative targets (or target ranges) for the inflation rate over one or more time horizons, and by explicit acknowledgment that low, stable inflation is the primary long-run goal of monetary policy. Among other important features of inflation targeting are vigorous efforts to communicate with the public about the plans and objectives of the monetary authority, and, in many cases, mechanisms that strengthen the central bank’s accountability for attaining those objectives”<sup>9</sup>.

Inflation targeting has received much attention recently from both academics and policymakers. This section first presents the achievements of inflation targeting. This is followed by the description of the framework of inflation targeting, the institutional and the operational framework. The section ends with a discussion of the adoption of inflation targeting in transition economies.

### 2.3.1 Achievements of Inflation Targeting

The aim of this section is to shed light on the accomplishments of inflation targeting as a new monetary policy regime. The main advantages of inflation targeting can be summarised as follows:

- Does not present the drawbacks of alternative monetary policy regimes<sup>10</sup>.
- Provides a nominal anchor for policy and the economy. The absence of other nominal anchors leads to improved credibility. Also setting a target helps in improving credibility.

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<sup>9</sup>Bernanke, Laubach, Mishkin, Posen (1999), p: 4.

<sup>10</sup>Comprehensive surveys of the comparisons between inflation targeting and alternative monetary regimes are McCallum (1995) and Cukierman (1995).

- Deals with time inconsistency problem.
- Reduces output volatility.
- Improves communication between policymakers and the public.
- Provides increased discipline, accountability and therefore improves credibility.

The success of inflation targeting started with the prevention of the incoherence that characterized monetary policy practice and analysis after the breakdown of the Bretton Woods regime system. This was subject to dramatic speculative attacks (Sweden, Finland, UK) and failed to achieve low and stable inflation. Trying to achieve the conditions for economic growth (price stability), over time, central banks have used *intermediate policy targets* for the exchange rate and various monetary aggregates. Thus, they were aiming to obtain the goal of price stability in an indirect way. At the beginning of 1990, the possibility of obtaining this objective in a direct manner highlighted the importance of inflation targeting. In order to influence inflationary expectations, central banks announced the future evolution of key nominal variables, known as *intermediate targets*, limiting their basic incentive to exploit short-run trade-offs between output and inflation (avoiding inflation bias). This consequently improved credibility. The significance of the intermediate targets is related to the fact that they act as nominal anchors. This prevents domestic or external shocks from increasing inflation above certain levels and generates a much more concrete long-term commitment to price stability. Intermediate targeting engages a preannounced **exchange rate rule** or targets for any particular monetary aggregate. Directed only at the exchange rate, monetary policy is seriously limited as it is constrained in responding to domestic or external shocks in contrast to the case of flexible exchange rate settings.

- *Developing and transition countries* that used a fixed exchange rate as a nominal anchor following the breakdown of the Bretton-Woods arrangement like those mentioned above, had to move to either more flexible arrangements or to the other extreme of currency board or full-fledged dollarization (Bulgaria, Estonia, Lithuania). This was due to the growing integration of world capital markets in the last twenty years and the increased volatility of capital flows, since the

EMS<sup>11</sup> and the Asia and Latin America crises (Grenville (2000)).

- *Developed countries* supported more flexible exchange rate arrangements choosing various monetary targeting regimes. Under these regimes, referred to as monetary targeting, monetary aggregates, considered as main determinants of inflation targeting in the long run, become the intermediate target. Under the context of monetary targeting, the main goal of central banks is to use all available instruments to control monetary aggregates. Hence, controlling the monetary aggregate is equivalent to stabilizing the inflation around the target. The success of monetary targeting as an intermediate target depends on the stability of its relationship with both the goal variables and the instrument used. Financial institutions developed money substitutes and the demand for money became unstable during 1980's. Although in the long run there is a stable relation between money and inflation, in the short-run they are not sufficiently correlated (Svensson, 1999). Therefore several OECD countries adopted explicit inflation targeting as a strategy for conducting monetary policy (Cukierman (1995), Croce and Khan (2000), Grenville (2000)).
- Additionally, inflation targets play the role of a *nominal anchor* for monetary policy providing a focus for expectations of the private sector and a reference point for central banks against which they can judge the desirability of short-run policies. A monetary policy is more effective in the presence of a firmly established nominal anchor and the more that anchor is understood by the public the better (Bernanke, Laubach, Mishkin and Posen (1999)). Under an inflation target framework, the monetary authority can conceivably pursue additional objectives but only to the extent to which they are consistent with the inflation targeting. The experience of the inflation targeters clearly shows that the coexistence of multiple anchors (typically, a crawling currency band and an inflation target, or a monetary aggregate target together with an inflation target) sooner or later is a source of potential policy conflict, which usually damages policy credibility.

It has been found that discretionary monetary policy, free from the constraints of the gold standard and after that of the Bretton Woods system of pegged exchange

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<sup>11</sup>European Monetary System.

rates, generates inflation and that unfettered discretion has not been a success. No rule describes how policy should be set in all possible outcomes, so some discretion is inevitable. But according to King (1999), that discretion must be led or constrained by a clear objective. Under inflation targeting, the long run objective of monetary policy is low inflation, therefore, this policy has been described as “constrained discretion” by Bernanke and Mishkin (1997). As discussed in Bernanke, Laubach, Mishkin and Posen (1999), inflation targeting combines some of the advantages traditionally ascribed to rules with those ascribed to discretion by imposing a conceptual structure and its inherent discipline on the central bank, but without eliminating all flexibility. An analytical distinction between policy rules and discretion can be drawn from the time-consistency literature<sup>12</sup>. Goodhard and Vinals (1994), Leiderman and Svensson (1995), Haldane (1995), McCallum (1996), Bernanke and Mishkin (1997) advocate the idea of inflation targeting as a framework for policy as opposed to a rule, based on earlier experiences in this area.

Moreover, Friedman and Kuttner (1996), have criticized inflation targeting first because they believe that it imposes a rigid rule on monetary policy and does not allow them enough discretion to respond to unforeseen circumstances<sup>13</sup>. It is also criticised for being an ‘inflation only’ scheme, targeting and ignoring output and unemployment. On the other hand, Debelle (1999) points out that the inflation targeting framework is sufficiently flexible to allow for a short-run trade off between output and inflation while at the same time maintaining medium-term price stability. He concludes that output stability has a definite role to play in inflation targeting. Svensson (1998) argues that inflation targeting is more than using a reaction function that responds in deviation between inflation and its target. First, because the central bank uses much more information in setting its instrument and secondly because targeting only one variable is inefficient. He states that it is better for the central bank to respond to both inflation and output by having a ‘flexible’ rather than ‘strict’ inflation targeting. Inflation targeting seems to be a commitment to a systematic and rational monetary policy, better than any other monetary regime so far, stabilizing not only inflation but output as

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<sup>12</sup>Kydland and Prescott (1977), Barro and Gordon (1983).

<sup>13</sup>This criticism has featured prominently in the rules-versus-discretion debate.

well. According to analytical studies on inflation targeting of Cukierman (1995) and Svensson (1996, 1998, 1999). An ideal policy rule (encapsulated in reaction function of the model) captures the central bank's general preferences in terms of bringing inflation to target within an acceptable time frame, without causing unacceptable deviations in output from potential or unacceptable movements in the monetary instruments. Debelle and Fisher (1994), provides evidence that there is a positive relationship between output and inflation variability, its strength depending on the weight put on output stabilization in the objective function. Svensson (1996) points out that the weight on output stabilization in the central bank's loss function is directly related to the rate at which inflation is adjusted towards the inflation target. With a positive weight, the inflation rate is adjusted gradually to the inflation target, having at a slower rate a larger weight, sustaining the results of Batini and Haldane (1999), and Debelle (1999).

Debelle (1999) evaluates the performance of different monetary policy rules in terms of their effect on inflation and output. He concludes that the Taylor rule outperforms other monetary policy rules regarding the reduction of both inflation and output variability (nominal income level rule, nominal income growth rule, price level rule, Taylor rule, inflation-only rule, change rule). For this reason, attempts to achieve an improved Taylor rule have been made, including forward-looking variables, exchange rate or forecast values of output and inflation. An extended discussion of monetary policy rules including the Taylor rule was presented in the previous section of this chapter.

Providing a nominal anchor for policy and the economy and dealing with the time inconsistency problem are major changes in the way the conduct of monetary policy takes place. Another aspect in which inflation targeting is changing direction is *credibility*.

Returning to the time consistency issue articulated by Kydland and Prescott (1977), Barro and Gordon (1983) and Calvo (1978), there is a suggestion that central bank secrecy may have a motive: to evade political pressure to pursue over-expansionary monetary policy that aims to exploit the short-run trade off between employment and inflation.

If the central bank is independent of the political process there are two approaches to this problem.

1. Appointing conservative central bankers, as suggested by Rogoff (1985)<sup>14</sup>, is a way of tackling this problem.
2. Another solution is to focus on the long run and “just do it”, as McCallum (1995) proposed.

As seen so far, starting with the implementation of inflation targeting in 1990, a different direction has been taken by central banks in tackling the time-inconsistency problem. It is acknowledged that a successful monetary policy relies heavily on the credibility of the central bank. Inflation targeting promotes a vast increase in transparency regarding inflation objectives which implies regular communication with the public.

Among the benefits of transparency achieved by explicitly announcing their objectives on the inflation front, the central banks:

1. Have been able to increase their credibility and anchor inflation expectations.
2. Were helped to achieve not only low and stable inflation but also decreased output volatility.
3. Gained increased public support.

However, things can be taken a step further, as Mishkin (2004) suggested, by raising the question of whether transparency will still be beneficial if it is pursued to a too high level. He argues that transparency is beneficial as long as it provides straightforward communication with the public and facilitates the central bank’s conduct of monetary policy optimally, through a proper focus on long-run objectives. Increased transparency can make the communication process more difficult and therefore reduce support for the central bank focus on its long-run objectives. This may occur especially if the central bank is announcing the objective function or projection of the path of the policy interest rate. Therefore, a problem that has to be solved is finding the optimal level of transparency for various conditions of the state of the central bank and/or the economy that can be met.

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<sup>14</sup>more weight is put on controlling inflation relative to output than does the general public, thus resisting inflationary policies.



Nevertheless, inflation targeting has the potential to serve two important functions which work together to improve the credibility of the central bank:

1. To improve communication between policymakers and the public.
2. To promote increased discipline and accountability for monetary policy.

Comprehensive surveys describing the practical experience of countries operating under inflation targeting (Leiderman and Svensson (1995), Haldane (1995), Almeida and Goodhart (1998)), provide evidence that in terms of communication, the announcement of inflation targets clarifies the central bank's objectives for markets and for the general public, reducing uncertainty about the future course of inflation.

However, although popular and successful, inflation targeting still brings issues that need to be discussed. Debates<sup>15</sup> around the topic of inflation targeting are related to the fact that in many countries, there has been a *shift in focus from how to achieve low inflation to how to conduct monetary policy in a low and stable inflation environment*, in 'an era of price stability' as Svensson (1999) stated. There is the question of whether the central banks benefit from greater operational flexibility or face new constraints in such an environment. Central banks have more freedom to concentrate on short-run economic problems without compromising long-run price stability. However, this may become more difficult as nominal interest rate come close to zero.

According to King (1999), the main problem is to *find the optimal inflation target* and to *reduce short-run output variability*. He is sceptical regarding the possibility of a short-run trade-off between inflation variability and output variability without compromising the long-run goal of price stability.

Fischer (1999) notes that despite the general agreement about the long-run strategy for monetary policy, much work needs to be done on *short-run policy implementation*. In particular he suggests that more effort should be directed to quantifying the short-run trade-off between inflation and unemployment and between inflation and the exchange rate, as well as the relationship of these trade-offs to the long-run goal of price stability.

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<sup>15</sup>New Challenges For Monetary Policy, A Symposium Sponsored By The Federal Reserve Bank of Kansas City, 1999.

The problem of *price-level versus inflation rate targeting* is widely discussed. Svensson (1999) gives consideration to price-level targeting as a potential substitute for inflation targeting, as suggested by Taylor (1999). Woodford (2003) agrees with this and furthermore, provides additional reasons for price-level targeting that might have theoretical and practical advantages over inflation targeting. King (1999) discusses the idea of price level targeting while suggesting that a price-level targeting or average inflation-rate targeting in which the price-level is stabilized around a trend might be more appropriate than a strict price-level target. Therefore, given these main debates summarized by King (1999), the objectives of monetary policy in a low inflation world are:

1. Finding the optimal inflation target.
2. Finding a way to stabilize output and unemployment.
3. Finding out whether to target prices or inflation.

Concerning the *first* objective, there are discussions regarding whether the target inflation rate should be zero or positive. The case of a zero inflation target discussed by Svensson (1999), King (1999), Metzler (1999), Mankiw (2001) is a significant problem, even though here there is also the problem of inflation measurement. This emphasises once again that when there is a high rate of inflation, the differences between methods of measuring inflation are not so significant but when the rate of inflation is low, they become more important. There are debates going on to determine whether there is a measure to fully capture relevant information to the monetary policy setting. However, the consensus is in favour of a small positive inflation rate. Targeting an inflation greater than zero allows the real interest rate to become negative, which offers the central bank the possibility of creating a negative interest rate as and useful tool for stimulating the economy in recession by stimulating spending. Evidence in this case is offered by Japan who has been stuck in a “liquidity trap”, while the monetary policy has had no effect. Mankiw (2001) argues in favour of a positive inflation rate, drawing attention not only to the fact that it avoids the liquidity trap but also because it “greases the wheels” of the labour market, as inflation makes labour markets work

better<sup>16</sup>.

There are two main reasons for the *second* objective. The first concerns the significance of downwards nominal rigidities in wages and prices (Yates (1998), King (1999)). The second concerns the fact that the nominal interest rate can not fall below zero. A fall below zero may constrain monetary policy in times of recession, resulting in a “liquidity trap” (King (1999), McCallum (2001)). Svensson (1999) argues that transparent inflation targeting that incorporates a moderately positive inflation target is the best way of avoiding this. Consequently, studies like Fischer (1993), Judson and Orphanides (1996), Hess and Morris (1996), Lucas (2000), Braumann (2000), whose study is based on the evidence of 17 countries, have emphasized the advantages of low inflation and the negative impacts of high inflation, in terms of low productivity, lower rates of growth and its costs. They all conclude that even a moderate rate of inflation is harmful to economic efficiency and growth. The aim of monetary policy is low and stable inflation in the long run. This is an important feature of economic growth.

### 2.3.2 Inflation Targeting’s Framework

As suggested by Masson, Savastano and Sharma (1997), Debelle, Masson, Savastano and Sharma (1999), Bernanke, Laubach, Mishkin, Posen (1999), Masson (1999), Mishkin (2000), Schaechter, Stone and Zelmer (2000), an approach to inflation targeting can be undertaken from the point of view of the frameworks involved, namely the *institutional* and *operational frameworks*. Therefore, aspects of the institutional framework, focusing on the legal framework of the central bank and the design of inflation targeting will be presented. This will be followed by a presentation of the operational framework and of issues regarding the actual implementation of inflation targeting.

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<sup>16</sup>Mankiw (2001), p:13. “The supply and demand for different kinds of labour is always changing. Sometimes an increase in supply or decrease in demand leads to a fall in the equilibrium real wage for a group of workers. If nominal wages can’t be cut, then the only way to cut real wages is to allow inflation to do the job. Without inflation, the real wage will be stuck above the equilibrium level, resulting in higher unemployment”.

## Institutional and Operational Frameworks

The institutional framework is characterized by two main points:

1. A central bank legal framework.
2. The design of inflation targeting

### a. Central Bank Legal Framework

*Firstly*, the main goal has to be *price stability* and *secondly*, the central bank has to be able to conduct monetary policy benefiting from *instrument independence* (central bank should be free to set its instruments).

- The *first* pre-requisite is a reflection of the fact that there is no long-run trade-off between output and inflation and that inflation is the only variable monetary policy can influence in long-run. Commitment to price stability is now seen as a key condition for achieving a broader economic stability (King, 1999), even though it is accompanied by fluctuations in output and unemployment.
- Regarding the *second* pre-requisite of the central bank's legal framework, as a practical aspect regarding the first requisites, according to DeBelle, Masson, Savastano and Sharma (1999), Mishkin (2000), no central bank is entirely independent of the influence of government, although it should have complete independence and exclusive control in setting its monetary instruments.

Political consensus in favour of a low inflation must be accomplished and price stability must be defined as the main goal of the central bank, although other goals of monetary policy (employment, output) may conflict with it. This needs legislative support for an independent central bank and there also has to be a clear distinction between politicians and board policymakers of the central bank. Subsequently, the monetary authority needs to have the ability and the willingness to *target only one indicator* (Masson, Savastano, Sharma (1997)).

### b. Design of Inflation Targeting

In order to implement inflation targeting, the framework of inflation targeting must be designed. This is realised by the determination of the target of inflation that is

going to be announced, the establishment of target horizons, price index, point range (or target range) and of the methods to prove and discuss the accountability and transparency methodology. Within a survey of the design of inflation targets in inflation targeting countries, a distinction has been drawn between those for developed and developing countries.

1. *Announcement of target* consists of establishing explicit quantitative targets for inflation targeting in the future and stating clearly to the public that inflation targeting takes precedence over all the other objectives of monetary policy. There also has to be communication with both people and government. It should be announced by the government or jointly by the government and the central bank.
2. *Target horizon* that consists in the period of time in which the inflation target is to be reached.
3. *Price index* - the consumer price index for most emerging market countries and core inflation for most developed countries. An important issue for discussion is the choice of the full inflation rate as a reference for the target and not core inflation. Choosing CPI instead of domestic inflation (GDP deflator), stabilizes both the domestic inflation and the real exchange rate.
4. *Formal escape clauses* - only used by certain countries (for example Brazil).
5. *Point range or target range* - target range is preferred by most countries given the uncertainties associated with hitting targets. Point targets have been adopted in some cases to focus inflation expectations.
6. *Accountability and transparency methodology* - implying press releases on policy changes, regular inflation outlook reports, active dialogue with the private sector, and in some cases, publication of inflation forecasting models. A final issue is the transparency of the inflation-targeting framework. As part of the initial setting, an effective communication process was established so that the population will be able to understand and monitor the decisions of the central bank and know the reasons why forecasted and accumulated inflation may deviate from the target.

The operational framework consists of:

- Quantitative economic models for inflation forecasting.
- Policy transmission channels.
- Policy implementation.

**Practical Aspects.** From the point of view of practical aspects, inflation targeting varies widely regarding its implementation features, including target price index, target width, target horizon, escape clauses, accountability of target misses, goal independence, and overall transparency and accountability regarding conduct of policy under inflation targeting.

The concept of full-fledged inflation targeting is based on five pillars:

1. An institutional commitment to price stability.
2. Absence of fiscal dominance.
3. Absence of other nominal anchors.
4. Policy instrument independence.
5. Policy transparency and accountability.

If these five conditions are satisfied, then there is effective conduct of monetary policy under inflation targeting. Some countries adopted inflation targeting without satisfying all five conditions. Brazil however adopted full-fledged inflation targeting from the beginning. The Bank of England started inflation targeting well before obtaining instrument independence, while Colombia, Israel, Korea, Mexico and South Africa have not published inflation forecasts (Christoffersen and Wescott (1999)). Regarding the assignment of the inflation targeting, Australia, Finland and Sweden's central bank did not have endorsement from the government when they announced the inflation target, while in Canada and New Zealand the target was the result of an agreement between the Ministry of Finance and the governor of the central bank. Regarding the inflation rate at the moment of adoption, the highest rate was registered by Chile (20%) followed by Israel (18%) and the lowest by Australia (1.8%).

The horizon of inflation targeting (which depends on the inflation rate at the time when the policy is adopted) varies within countries. In the disinflation process in Canada and New Zealand, the authorities specified an horizon of 18 months for reaching the target, reducing it to 12 months for New Zealand in the next step. The targets were set in both countries for 5 years from the moment when inflation was reduced. In Australia, the time horizon for the inflation targeting was the length of the business cycle. The success of New Zealand and Canada in adopting inflation targeting to achieve disinflation, convinced the UK, Finland, Sweden and Australia to adopt inflation targeting, although the inflation rate in these countries was not as high in comparison with New Zealand and Canada (Debelle, Masson, Savastano and Sharma, 2000). Regarding the specification of the width of the target band (because of the imperfect control of the monetary policy over the inflation rate), a narrow band usually means a strong commitment to the inflation target, as it has a close relation with credibility. Australia and Finland have focused on a particular point target for the inflation rate, while Canada, New Zealand, Sweden and UK specified a range for the inflation rate. The Spanish monetary authority specified the target in terms of a ceiling.

### **2.3.3 Adopting Inflation Targeting in Transition Countries - A Credibility Issue**

According to Taylor(1999) and Svensson (1997c), inflation targeting is implemented by adopting monetary-policy rules defined by Svensson (1999) as a “prescribed guide for monetary policy conduct”, on the way to achieve price stability. However, the implementation of inflation targeting is difficult due to the imperfect control of inflation, knowing that it reacts to changes in the monetary policy instrument with long and variable lags, since inflation is affected also by other factors than monetary policy. In transition economies, the length and the variability of these lags are bigger due to the specific economic circumstances and perhaps mainly because of lack of credibility. It appears that the adoption of inflation targeting in transition economies does not bring only advantages as there are also preconditions for the successful implementation of inflation targeting.

## Advantages and Disadvantages of Inflation Targeting

Comparative studies of inflation targeting and non-inflation targeting countries of Corbo, and Schmidt-Hebbel (2000) and Mishkin and Schmidt-Hebbel (2000) present the achievements recorded by the countries who adopted inflation targeting. Mishkin (2000), Mishkin and Schmidt-Hebbel (2000) and Debelle, Masson, Savastano and Sharma (1999) point out the advantages and disadvantages of implementing this framework in transition countries.

As regarding the *advantages* of inflation targeting Mishkin (2000) states that inflation targeting:

- Uses all the information to determine the best instrument for the monetary policy.
- Reduces the possibility of the central bank falling into the trap of time-inconsistency.
- Also sets the goals of the central bank in the long run regarding what it can do and not what it cannot (controlling inflation instead of raise output).

Masson, Savastano and Sharma (1997) state that inflation targeting helps in reducing sacrifice ratios and output volatility by guiding inflation expectations. Inflation targeting does not reduce inflation expectations quickly but gradually over time. It also strengthens forward-looking expectations on inflation, weakening the weight of past inflation inertia.

According to Mishkin (2000), four of the *disadvantages* of implementing inflation targeting in transition countries namely rigidity, too much discretion, the potential to increase output instability and the possibility of lower economic growth are not too troublesome, while three others provide more serious problems:

1. Due to the fact that inflation is difficult to control, the accountability of the central bank can be very weak, especially in cases where there are long lags from monetary policy instruments to the inflation outcome.
2. Inflation targeting cannot prevent fiscal dominance, as discussed by Masson, Savastano and Sharma (1997). Although there is an inflation targeting regime, it is not necessarily true that the fiscal policy will be an adequate one. If it is not, this will lead to high inflation. The absence of fiscal dominance is crucial for a successful inflation targeting strategy.



3. A third disadvantage is the instability brought by exchange rate flexibility, which is one of the requirements of inflation targeting.

Given that the results of changing the instrument settings can be seen only after a considerable lag, inflation forecast errors are significant and inflation targets tend to be loosened. In this case, it is problematic for the central bank to maintain the necessary credibility for an inflation targeting strategy. Therefore, in order to improve the credibility of the central bank and to make the successful implementation of inflation targeting smoother and faster, several preconditions must be fulfilled.

### **Preconditions for Implementing Inflation Targeting in Transition Economies**

According to Masson, Savastano and Sharma (1997), Debelle, Masson, Savastano and Sharma (1999), the preconditions for implementing inflation targeting in transition economies are:

1. First, the central bank has to be able to conduct monetary policy with some degree of independence. Institutional commitment to price stability is the primary goal of the central bank. Price stability must be the primary goal when there are conflicts with other goals. This needs legislative support for an independent central bank and also there has to be a clear distinction between politicians and board policymakers of the central bank.
2. The second requirement is that the monetary authority needs to have the ability and the willingness to target only one indicator. Here is all about credibility. If a country chooses to target for example the fixed exchange rate, all the other objectives are subordinated to the exchange rate, the policymakers being unable to operate an inflation target system. In this situation, the public has no assurance about which of them will receive precedence and neither of the two policies can have the needed credibility for success.
3. Third, there are the preliminary conditions a country needs to meet in order to be able to implement inflation targeting.
  - First, it has to establish explicit quantitative targets for inflation targeting some periods ahead.

- Second, to state clearly to the public that inflation targeting takes precedence over all the other objectives of monetary policy.
- There also has to be communication with people and frequent communications with the government.
- Moreover a model or methodology has to set up for inflation forecasting and to have a forward looking operating procedure in which monetary policy instruments are adjusted in order to hit the chosen target.
- One of the requirements of inflation targeting is a flexible nominal exchange rate and for this, fluctuations are inevitable. Developing countries have to be careful to subordinate this problem to the inflation objective in order to obtain a successful inflation-targeting regime.

Smidkova and Hrnčir (2000) divided the factors that determine a successful implementation of inflation targeting in a transitional country in three categories:

1. Preconditions: preference for price stability, an adequate macroeconomic and institutional framework, sufficiently accumulated know-how of the central bank.
2. Domestic constraining factors: an insufficient legal framework, incomplete information on the medium term reform strategy.
3. External constraining factors: international financial turbulence, large changes in world prices.

As can be seen, one of the main issues in adopting inflation targeting is the lack of credibility without which inflation targeting can not be successful. However, helped by the initial implementation of the necessary preconditions, credibility is gained over time through committed and consistent policy.

## 2.4 The Conduct of Disinflationary Monetary Policy Under Imperfect Credibility and the Optimal Speed of Disinflation

One of the aims of this thesis is to discuss the effects of imperfect credibility on output and on the optimal speed of disinflation in a transition to price stability. Two main areas of debate are of interest for this thesis.

First, Ball (1994) presented a result which generated a great deal of debate. He demonstrated that a contractionary monetary policy with staggered price settings can give rise to a boom in output in conditions of perfect credibility (rational expectations). In data, this has hardly been the case. Therefore, much attention has been given to this result which needs to be elucidate. Indeed, with the onset of the rational expectations revolution, an increased interest in the credibility of monetary policy, to which this is in part related, started to occur. As mentioned earlier, under the assumption of rational expectations, a completely credible central bank can engineer a disinflation without any adverse effects on output. But central bank credibility still matters even though expectations are less than fully rational.

Second, the New-Keynesian sticky price framework appears to be empirically ineffective in generating the adequate inflation and output persistence. This has been addressed by Phelps (1979), Taylor (1983), Ball (1994), Fuhrer and Moore (1995).

A set of innovations has been forthcoming attempting to develop the New Keynesian framework so that it offers a better empirical fit and to address Ball's problem. One of them is the implementation of imperfect credibility and learning (Andolfatto and Gomme (1999), Hugh and Lansing (2000), Erceg and Levin (2001)). This chapter will present a review of some of the literature concerned with study of the effects of a disinflationary monetary policy when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price-level, under both perfect and imperfect credibility. A disinflationary monetary policy like the one mentioned above, under perfect credibility is not uncommon in the literature (Ball (1994), Ireland (1997), King and Wolman (1999) and Khan, King and Wolman (2003)). The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices.

Recent research has revealed much about the effects of monetary contraction in such an environment. For the purpose of this research, three broad results stand out from this work and are going to be the focus of this thesis.

1. First, in the periods following a contraction in the money stock, real output is likely to fall below its (now altered) long-run equilibrium level.
2. Second, a gradual disinflation may actually result in output, after its initial decline, rising above its new steady-state level and remaining there for some time.
3. Finally, it is optimal to end high inflations quickly, low inflations gradually, and maintain inflation at or near zero, thereafter.

Important precursors to the analytical foundations of these results are contained in Danziger (1988), Benabou and Konieczny (1994) and Lucas and Stokey (1983), while the contributions of Sargent (1982) and Gordon (1982), as emphasized by Ireland (1997), provide an important focus on policy implications of differential speeds of disinflation.

Ireland (1997), as many others, assumes perfect foresight (or rational expectations). This assumption is appropriate when one wishes to analyse an economic model in, or in a close neighbourhood of, an unchanging steady state. However, this assumption of perfect foresight may be less attractive when one wishes to analyse the effects of 'large' changes in policy. This is due to the reason that the steady state of the model may well be changing and, in addition, policymakers may not enjoy complete credibility. However, it would be interesting to examine the effects of a disinflationary monetary policy when policymakers initially do not enjoy complete credibility, and where the steady state of the economy is changing.

Two important recent contributions address some of these issues. The first is Ball (1995). He demonstrates that if credibility is sufficiently low, a period of disinflation may lead to expected output losses. In his model agents harbour a nagging suspicion that the authorities will renege and give up on the path of disinflation. He models agents' scepticism as a constant conditional probability of renegeing. This may be a somewhat rigid way of modelling the evolution of agents' priors.

This way of modelling imperfect credibility is not uncommon in the literature. For example, Almeida and Bonomo (2002) using endogenous state-dependent rules to examine output costs of monetary disinflation, treat briefly the case of imperfect credibility modelled in this manner. This is also employed by Bonomo and Carvalho (2004) who examine the output effects of monetary disinflation in a model with endogenous time-dependent pricing rules and imperfect credibility of the disinflation policy. They find that these features interact to generate an additional effect on top of the ones obtained with either endogenous time-dependent rules (Bonomo and Carvalho (2004)) or imperfect credibility (Ball (1995)) which consists in higher output costs of monetary disinflation.

However, although popular, this manner of modelling imperfect credibility is not the most appropriate, as it may be a somewhat rigid way of modelling the evolution of agent's priors. This is due to the fact that on one hand, as the disinflation proceeds, it is plausible that agents accord increasing weight to the announced path for the money supply. On the other hand, perhaps as the disinflation proceeds and the extent of nominal rigidity in the economy optimally rises, the authorities may be more likely to renege (to exploit a flattening of the Phillips curve). It can be argued that both these cases seem intuitively plausible. Therefore, this thesis suggests and implements a new way of modelling this, by adopting an 'expectations updating rule' that nests both these alternatives, as the distinction between these two cases can be important. This will also allow the study of the impact of a temporary lack of credibility during a disinflationary policy.

The second important recent contribution is by Ireland (1995). He also finds that higher output losses are the price of imperfect credibility during a period of disinflation. However, the attainment of price stability is desirable (i.e., welfare enhancing) in general, except when the loss of seigniorage is replaced in the low inflation state by a rise in other distortionary taxes. Once again, his modelling of the expectations formation process misses the effects mentioned above.

Moreover, Ireland (1997) calculates of the optimal speed of disinflation for the case of perfect foresight. He reconciles two major contributions to the literature, Sargent (1982) and Gordon (1982), which express two opposite views on this issue. Gordon (1982) states that disinflation should be done gradually in order to be realised with

minimal output loss, while Sargent (1982) argues that only by proceeding to immediate disinflation, can the output loss is minimized. Ireland (1997) reconciles these two views, stating that small inflations should be stopped gradually, while big inflations immediately. Extending Ireland's (1997) calculation of the optimal speed of disinflation to the case of imperfect credibility appears to be an interesting research question. It can be enquired whether or not imperfect credibility materially impacts on the optimal speed of disinflation, as compared to perfect foresight.

## 2.5 Conclusions

The literature reviewed for the purpose of this thesis discussed three main areas. These areas spoke to the issue of imperfect credibility in monetary policy. The first topic discussed was that of monetary policy rules which when implemented, may improve the credibility and the effectiveness of monetary policy. The second related topic is that of inflation targeting.

The third topic, moving towards a more detailed analysis of an economy, discusses some of the important recent contributions regarding the conduct of a disinflationary monetary policy. The focus is on the behaviour of output and on the optimal speed of disinflation in conditions of imperfect credibility.

Two main conclusions arise from this literature review. The first conclusion is with regard to the impact the implementation of monetary policy rules and inflation targeting has in addressing the problem of imperfect credibility. The second, which lead to the identification of the research questions to be explored in this thesis, is that although research on the disinflationary monetary policy under imperfect credibility has been conducted, little has been said about the behaviour of output in an environment in which there is a temporary lack of credibility. In addition, there is needed a more flexible way of modelling imperfect credibility than so far seen in the literature. Also the behaviour of the optimal speed of disinflation under a temporary lack of credibility has not been explored so far in the literature and is a topic discussed in this thesis.

# Chapter 3

## The Model

### 3.1 Introduction

Chapter 2 of this thesis consists of a review of the literature concerning imperfect credibility. The first section discusses monetary policy rules as a remedy for imperfect credibility, while the second section is dedicated to inflation targeting, a monetary policy regime meant to address the drawbacks imperfect credibility holds on the conduct of monetary policy. The final section presents a discussion of the literature which analyses a disinflationary monetary policy when the policymakers are committed to price stability, emphasizing the effect of imperfect credibility on output and also on the optimal speed of disinflation. Therefore, an interesting aspect to examine is the impact of a temporary lack of credibility during a transition to price stability in terms of an actual economy. More specific, to see how costly this would be in terms of output loss. To explore this, a New Keynesian model with sticky prices as in Ireland (1997), King and Wolman (1999), Khan, King and Wolman (2003), has been employed. This will be the 'backbone' of this thesis having been adopted to study the impact of a temporary lack of credibility in a transition to price stability on the output (Chapter 6) and on the optimal speed of disinflation (Chapter 7).

The model, among whose special features is that it combines time dependent strategies with state dependent strategies, allows to study the effects of a disinflationary monetary policy on output and the optimal speed of disinflation when policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price-level. Important precursors of the analytical foundations of these results are contained in Danziger (1988), Benabou and Konieczny (1994), Lucas and Stokey (1983), while the contributions of Sargent (1982) and Gordon (1982), as emphasized by Ireland

(1997), provide an important focus on policy of differential speeds of disinflation.

The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices. Recent research has revealed much about the effects of monetary contraction in such an environment. Key papers that develop results based on this class of model are Ball (1994), Ireland (1997), King and Wolman (1999) and Khan, King and Wolman (2002). However, no work has been done so far to explore the effects of imperfect credibility in a framework like the one described by this model. This will be the focus of this thesis. More specifically, the effect of a temporary lack of credibility on output and optimal speed of disinflation will be studied. This has been realised by bringing an innovation to the model in order to incorporate the case of imperfect credibility. This will be presented in more depth in Chapter 4.

The outline of the chapter is as follows: Section 3.2 presents the representative agent side of the model, followed by the corporate sector in section 3.3, which describes the single and two price strategies employed by the agents in setting their prices and also in steady state conditions. The disinflation policy employed by the authorities is described in section 3.4 and the parametrization of the model in section 3.5.

## 3.2 The Representative Agent

The economy consists of many identical consumers. In each period a representative agent makes plans for consumption and leisure/labour so that (expected) present discounted value of utility is maximized.

It is assumed that the current period utility function of the representative household takes the iso-elastic form,

$$U_t(C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma N_t \right\} \quad \alpha, \gamma > 0, \quad (3.1)$$

where,  $U_t : R_+ \times R_+ \rightarrow R$ , is separable in consumption,  $C_t$ , and labour supply,  $N_t$ .  $\beta \in (0, 1)$  is a discount factor. Following Dixit and Stiglitz (1977),  $C_t$  is defined over a continuum of goods:

$$C_t = \left[ \int_0^1 c_t^{\frac{b-1}{b}}(i) di \right]^{\frac{b}{b-1}} \quad b > 0, \quad (3.2)$$

where  $c_t(i)$  denotes, in equilibrium, the number of units of each good  $i$  from firm  $i$



that the representative agent consumes.  $b$  is the price elasticity of demand.  $p_t(i)$  is the nominal price at which firm  $i$  must sell output on demand during time  $t$ . The Dixit-Stiglitz aggregate price level  $P_t$  at time  $t$  is then given by:

$$P_t = \left[ \int_0^1 p_t^{1-b}(i) di \right]^{\frac{1}{1-b}}. \quad (3.3)$$

Let  $N_t$  be given by:

$$N_t = \int_0^1 n_t(i) di,$$

where  $n_t(i)$  denotes the quantity of labour supplied by the household to each firm  $i$ , at the nominal wage  $W_t$ , during each period. This assumption means that households effectively supply a portion of labour to *all* firms. The reason why such an assumption is needed (and the one below regarding the representative agent's share portfolio) is to ensure that the marginal utility of wealth equalizes across agents.

Each period, the representative agent faces a budget constraint of the following sort:

$$\int_0^1 [Q_t(i) s_{t-1}(i) + \Phi_t(i) + W_t n_t(i)] di \geq \int_0^1 [P_t(i) c_t(i) + Q_t(i) s_t(i)] di. \quad (3.4)$$

Here  $Q_t(i)$  denotes the nominal price of a share in firm  $i$ ,  $s_t$  denotes the quantity of shares,  $\Phi_t(i) di = D_t(i) s_t(i)$ , where  $D_t(i)$  is the dividend associated with a unit share, and  $\int_0^1 P_t(i) c_t(i) di = P_t C_t$  denotes total nominal expenditure. It is assumed that for  $t = 0$ ,  $s_{-1}(i) = 1$ , for all  $i \in [0, 1]$ . In effect, then, it is assumed that each household owns an equal share of all the firms for each good  $i$ . The constraint (3.4) says that each period (and, under uncertainty, in each state of nature) income (financial plus labour) can be worth no less than the value of expenditure (on non-durable consumption plus financial investment). The household problem is to choose  $c_t(i)$ ,  $n_t(i)$ ,  $s_t(i)$  and aggregate consumption  $C_t$ , so as to maximize (3.1) subject to the sequence of constraints (3.4) and the relevant initial and transversality conditions. Optimal household behaviour is described by the requirement that household consumption spending must be optimally allocated across differentiated goods at each point in time (i.e., the optimal  $c_t(i)$ ). It can be shown therefore, that the Dixit-Stiglitz preference relation requires

that purchases of each good  $i$  satisfies:

$$c_t(i) = C_t \left( \frac{p_t(i)}{P_t} \right)^{-b}. \quad (3.5)$$

As in Ireland (1997) it will simplify things somewhat if the aggregate nominal magnitudes are determined in equilibrium by a quantity-type relation:

$$M_t = \int_0^1 P_t(i) c_t(i) di = P_t C_t. \quad (3.6)$$

An interior optimum for the agent's problem will include (3.4) with equality, (3.5) for all  $i$ , (3.6) and the following conditions:

$$C_t^{1-\alpha} = \lambda_t P_t, \quad (3.7)$$

$$\gamma = \lambda_t W_t. \quad (3.8)$$

And for all  $i$

$$Q_t(i) = D_t(i) + \beta E_t(\lambda_{t+1}/\lambda_t) Q_{t+1}(i), \quad (3.9)$$

where  $\lambda_t$  is unknown multiplier associated with (3.4).

We denote:

$$\frac{M_t}{M_{t-1}} = \theta_t, \quad (3.10)$$

as the rate of nominal money growth. The level of real balances is given by:

$$\frac{M_t}{P_t} = Y_t.$$

In equilibrium, the aggregate consumption  $C_t$  must at all times equal the level of real balances.

$$C_t = Y_t. \quad (3.11)$$

### 3.3 The Corporate Sector

There is a continuum of firms indexed by  $i$  over the unit interval, each of them producing a different, perishable consumption good. So, goods may also be indexed by  $i \in [0, 1]$ , where firm  $i$  produces good  $i$ .

Each firm  $i$  sells shares, at the beginning of each period  $t$ , at the nominal price  $Q_t(i)$ ,

and pays, at the end of the period, the nominal dividend  $D_t(i)$ . The representative household trades the number of shares that it owns,  $s_t(i)$ , in each of the firms, at the end of each period  $t$ . Under market clearing,  $s_t(i) = 1, \forall i \in [0, 1]$ , in each period. Firms are able to change prices each period, subject to a fixed cost. As a consequence, in equilibrium firms will not necessarily be willing to change prices in each period. The criterion for a price-setting decision at time  $t$  is to maximize the returns to the shareholders.

At time  $t$  it is assumed that firms are divided into two categories, so that for  $t = 2l, l \in Z_+$ , firms from the first set can freely change their price for output,  $p_{1,t}(i)$ . The firms belonging to the second set, meanwhile, must sell output at the same price set a period before,  $p_{2,t}(i) = p_{2,t-1}(i)$ , unless they pay the fixed cost  $k > 0$ , measured in terms of labour. This cost may be thought of as being associated with information collection and decision making. For  $t = 2l + 1$ , the roles are reversed and the first set of firms keeps prices unchanged,  $p_{1,t}(i) = p_{1,t-1}(i)$  unless they are willing to pay the fixed cost  $k$ , while the second set of firms, can freely set new prices.

The model assumes that firms are constantly re-evaluating their pricing strategy, weighting the benefits of holding prices fixed versus the alternative of changing prices and incurring the fixed penalty. However at moment  $t$  the firms belonging to the set of firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At more moderate rates of inflation or in the face of gradual changes in the monetary stance, they are more likely to keep their prices constant for two periods and hence avoid the cost  $k$  (single price strategy). On the other hand, in the case of high inflation or in the face of sharp changes of the monetary stance, firms are more likely to choose a new price and pay the cost  $k$  (two price strategy).

A simple linear production technology is assumed  $y_t(i) = l_t(i)$  where  $y_t(i)$  and  $l_t(i)$  are output of firm  $i$  and labour used to produce it, respectively. Let us denote aggregate output as  $Y_t$ , then equilibrium profits at time  $t$  for firms  $i$  are given by:

$$D_t(i) = [p_t(i) - W_t] \left( \frac{p_t(i)}{M_t} \right)^{-b} C_t - I_t(i)W_t k. \quad (3.12)$$

While, in equilibrium, the units of labour supplied to each firm at nominal wage

$W_t$  are given by:

$$n_t(i) = Y_t^{1-b} \left( \frac{p_t(i)}{M_t} \right)^{-b} + I_t(i)W_t k,$$

$$\text{where } I_t(i) = \begin{cases} 0, & \text{if the firm pays the cost of price adjustment } k \text{ at moment } t; \\ 1, & \text{if the firm does not pay the cost } k \text{ at moment } t. \end{cases}$$

### 3.3.1 Single price strategy

Under this strategy, we may think of firm  $i$  choosing  $p_t(i)$  so as to maximize the following expression:

$$\Pi(i)_t = D_t(i) + \beta E_t (\lambda_{t+1}/\lambda_t) D_{t+1}(i), \quad (3.13)$$

which follows from (3.9) and implies that prices are set to maximize market value. It is set  $\gamma = 1$ , substitute (3.7), (3.8), the quantity equation and goods market equilibrium into (3.12). It then follows that the price of firm  $i$  that will be used for two consecutive time periods is:

$$p_t(i) = \frac{b}{b-1} \frac{M_t^b Y_t^{b-1} + \beta E_t M_{t+1}^b Y_{t+1}^{b-1}}{M_t^{b-1} Y_t^{b-\alpha} + \beta E_t M_{t+1}^{b-1} Y_{t+1}^{b-\alpha}}. \quad (3.14)$$

This equation is familiar to the New Keynesian economics. It states that the optimal price will be a function of current and future anticipated demand and costs conditions and where in a steady state price will be a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition based on Dixit-Stiglitz preferences, the mark-up is constant and determined by the elasticity of demand (that is, tied down via the preference side of the model), the lower the elasticity, the higher the mark-up.

### 3.3.2 Two price strategy

In this case, the firm chooses the price  $p_t(i)$  to maximize profits in each period:

$$\Pi_t(i) = D_t(i). \quad (3.15)$$

The optimizing price in this case is given by:

$$p_t(i) = \frac{b}{b-1} \frac{M_t}{Y_t^{1-\alpha}}. \quad (3.16)$$

Here it can be seen that prices are marked-up as before; now only current period

demand and cost conditions that are relevant.

### 3.3.3 Steady-State

In steady-state, money growth is constant,  $\theta_t = \theta$ , and consequently, real money balances  $Y_t = Y$ , are constant. From equation (3.11), it is shown then that  $C_t = Y_t = Y = C$ , so that aggregate output is constant as well. Similarly, it can be seen that  $N_t = N$ , any  $t$ .

In steady state firm  $i$  may optimally choose its price strategy, either the one-period or the two-period strategy (depending on inflation). Furthermore, these steady state corporate decisions will be set out, along with those for aggregate output and aggregate employment for each of the two strategies.

#### Single price strategy

Using equation (3.14) the steady state price under the single-price strategy is:

$$p(i) = \frac{b}{b-1} \frac{1 + \beta\theta^b}{1 + \beta\theta^{b-1}} \frac{M_t}{Y^{\alpha-1}}. \quad (3.17)$$

The first set of the firms establish new prices at  $t = 2l$ , while the second set of firms fix new prices at  $t = 2l + 1, l \in Z_+$ . Hence, using the suitable relation of the price for each of the two strategies, it follows that the steady state aggregate output is:

$$C = \left( \frac{b}{b-1} \frac{1 + \beta\theta^{b-1}}{1 + \beta\theta^b} \left( \frac{2}{1 + \theta^{b-1}} \right)^{\frac{1}{1-b}} \right)^{\frac{1}{\alpha}}.$$

Aggregate employment is then given by:

$$N = C^{1-\alpha b} \frac{1 + \theta^b}{2} \left( \frac{b}{b-1} \frac{1 + \beta\theta^{b-1}}{1 + \beta\theta^b} \right)^b.$$

#### Two price strategy

Under the two-period strategy the steady state price for good  $i$  is:

$$p(i) = \frac{b}{b-1} \frac{M_t}{Y}. \quad (3.18)$$

Steady state aggregate output:

$$C = \left( \frac{b-1}{b} \right)^{\frac{1}{\alpha}}.$$

In this case, aggregate employment is greater than the steady state value of the output due to the fixed cost of price adjustment paid by half of all firms in the economy.

$$N = \left( \frac{b-1}{b} \right)^{\frac{1}{\alpha}} + \frac{k}{2}.$$

### 3.4 Disinflation Policy

This section presents the sort of process for the money growth rate employed by the authorities to achieve disinflation over some horizon. This is the approach adopted in Ireland (1997), following Ball (1994).

Specifically, at period 0, the authorities announce a path for the money supply,  $\{M_t^A\}_{t=0}^T$ , so that by time period  $T$  inflation will be zero. The superscript  $A$  indicates the 'announced' level of the money supply. This announced path for the money supply, in turn, implies a gradual decrease in the growth rate of the money supply.

According to (3.10),  $\theta_t$  denotes the growth rate of the money stock at time  $t$ . In this work there are studied processes for the money growth rate of the following type:

$$\theta_t = \theta_{t-1} - \frac{\theta - 1}{T}, \quad (3.19)$$

for any value of  $t$  from 0 to  $T - 1$ , where  $\theta_{-1}$  is equal to the initial rate of inflation, and where  $\theta_{t>T} = 1$ . So, a horizon of time  $T = 1$  entails immediate disinflation, while for  $T > 1$  the policymakers engineer a more gradual path towards price stability.

### 3.5 Parametrization of the model

The calibration of the model follows Ireland (1997). He follows the parametrization derived by Ball, Mankiw and Romer (1988) for the intertemporal elasticity of substitution  $\alpha = 0.1$  and Rotemberg and Woodford (1992) for  $b = 6$ , which corresponds to a benchmark value of 1.2 for the steady-state mark-up. Following Ball and Mankiw (1994), each interval of time in the model corresponds to a period of six months, determining the choice of  $\beta = 0.97$ , which is consistent with an annual discount rate of

5%.  $k$ , the inflation rate for which the rigidity of individual goods vanish and firms switch from the single price strategy to the two price strategy, is chosen at the value of 0.1075. The values of the parameters used by Ireland (1997) and implicitly in this model,  $\alpha, \beta, b, k$ , correspond to an acute case of disinflation. Finally, it is assumed  $\gamma = 1$ .

### 3.6 Conclusions

The New Keynesian model with sticky prices presented in this chapter has been employed by Ireland (1997), King and Wolman (1999), Khan, King and Wolman (2003). It has also been adopted in this work to study the impact of a temporary lack of credibility in a transition to price stability on the output (Chapter 6) and on the optimal speed of disinflation (Chapter 7).

The reason behind this choice is twofold. First, it allows to study the effects of a disinflationary monetary policy on output and the optimal speed of disinflation when policy makers are committed to achieve price stability. The analysis takes place in an environment where the supply-side of the economy is characterized by monopolistically competitive firms, and where there is rigidity in the setting of prices. Second, although recent research has revealed much about the effects of monetary contraction in such an environment, no work has yet been done employing this model to examine the effects of temporary lack of credibility on either output or the optimal speed of disinflation.

The manner in which imperfect credibility and implicitly a temporary lack of credibility have been modelled and implemented in this model will be presented in Chapter 4. Chapter 5 will discuss the methodology employed for solving the new model.

# Chapter 4

## Disinflation under Imperfect Credibility

### 4.1 Introduction

As discussed in Chapter 2, the assumption of perfect foresight may be less attractive when one wishes to characterize the path of output in a transition to price stability. Therefore, the current chapter presents a way imperfect credibility can be modelled in order to characterize the path of output in a transition to price stability for the case of imperfect credibility.

Moreover, imperfect credibility has been modelled to incorporate the missing effects from the way imperfect credibility had been previously modelled by Ball (1995) and also Ireland (1995). As presented in section 2.4 of this thesis, Ball (1995) models agent's scepticism<sup>1</sup>, in a way considered relatively rigid. In this thesis are considered two cases in which agents attach increasing or decreasing weight to the announced path of the money supply.

Temporary imperfect credibility is modelled by introducing a way to capture the movement of the probability mass through time towards the central's bank announced money supply path. The evolution of the probability mass, which characterizes the agent's subjective expectations, is realized by implementing a learning process. What is meant by 'learning' in this thesis will be specified in detail in this chapter. Moreover, there will be three different exogenous types of learning employed. Each of them captures different evolution paths describing the willingness of the private sector to believe the government's announcement about the money supply after the change in

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<sup>1</sup>as a constant conditional probability of reneging



policy. There will be also an attempt to measure the degree of lack of credibility in the government's announcement during a disinflationary process.

The outline of the chapter is as follows: Section 4.2 presents the way imperfect credibility is modelled. This section specifies the expectation updating rules employed in 4.2.1 and the concept of 'learning' applied in this thesis in 4.2.2. Section 4.3 discusses the types of learning considered, starting with linear learning in 4.3.1. The debate concerning the case for non-linear learning is found in 4.3.2, followed by the two types of non-linear learning: concave in 4.3.3 and convex in 4.3.4. Subsection 4.3.5 presents the two types of non-linear learning in a nested process. Section 4.4 presents the way the degree of lack of credibility is quantified over a disinflationary process and its behaviour under each of the types of learning considered: linear learning in subsection 4.4.1, concave and convex learning in 4.4.2 and 4.4.3 respectively. 4.4.4 comprises an analysis of the two non-linear types of learning in relation to the degree of lack of credibility. Conclusions are summarized in section 4.5. The chapter ends with an appendix which presents the solutions to some of the mathematics involved in sections 4.4.3-4.4.5.

## **4.2 Modelling Imperfect Credibility**

This section presents the manner in which imperfect credibility, which fades into perfect credibility due to learning, is modelled. This is achieved by defining how the private sector updates its expectations and by specifying what it is meant by learning and how it occurs.

### **4.2.1 'Expectation Updating Rules'**

The policy employed by the monetary authority is to lower money growth linearly to zero, over some time horizon,  $T \geq 1$ , according to (3.19). In this thesis, it is assumed that the private sector has its own expectations of the money supply and the level of the money supply employed in the model is given by an expectation updating rule. Regarding what the private sector is expecting to happen with the money supply in the future and in order to retain computational manageability, it is assumed that agents perceive only two possible outcomes:

1. The monetary authority's announced path for the money supply.

2. A reversion to an alternative, more inflationary path for the money supply.

Moreover, two obvious choices for the second alternative path have been modelled:

- (a) Agents perceive the authorities as reverting to the previous steady state inflation.
- (b) Agents fear that the government will 'run out of steam' so that at time  $t$  (for  $0 < t < T$ ) the growth rate of the money stock will be equal to the growth rate between  $t - 1$  and  $t$ .

These two possible outcomes are incorporated in the set-up of the 'expectation updating rules'. It is assumed that the expectations are formed as a weighted mean between the first possible outcome and each of the two alternatives of the second one.

The first option is when agents perceive the authorities as reverting to the previous steady state inflation. This is given by the previous period level of money supply, known by the private sector, adjusted by the initial rate of inflation. Algebraically, the expectation updating rule is as follows:

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{-1}M_{t+j-1} + (1 - \rho_{t+j})M_{t+j}^A. \quad (4.1)$$

The second option is when they fear that the government will 'run out of steam' such that at time  $t$  (for  $0 < t < T$ ) the growth rate of the money stock will be equal to the growth rate between  $t - 1$  and  $t$ . This is algebraically characterized as follows:

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{t-1}M_{t+j-1} + (1 - \rho_{t+j})M_{t+j}^A. \quad (4.2)$$

Furthermore, it will be assumed that the authorities in fact do stick to the announced path for the money supply, so in practice (4.1) and (4.2) may be rewritten as:

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{-1}M_{t+j-1}^A + (1 - \rho_{t+j})M_{t+j}^A, \quad (4.3)$$

or

$$E_{t+j-1}M_{t+j} = \rho_{t+j}\theta_{t-1}M_{t+j-1}^A + (1 - \rho_{t+j})M_{t+j}^A, \quad (4.4)$$

where  $\rho_t$  is the probability mass characterizing agent's subjective expectations, with  $t \in [0, T]$ , where  $T$  is, as mentioned before, the length of the interval of time over which the disinflation process takes place.

The case in which  $\rho_t = 0$ , for any  $t \in [0, T]$ , corresponds to absolute, perfect credibility of the private sector in the authorities' announcement, since the expected value of the money supply is the one announced by the monetary authority. In this circumstance, both 'expectation updating rules' presented nest the case of perfect foresight/credibility of Ireland (1997). In this way both alternatives of perfect and imperfect credibility are incorporated in one algebraic equation.

At this point, how imperfect credibility has been modelled by implementing an expectations updating rule has been discussed in some detail. Moreover, not only has imperfect credibility been modelled, but also a temporary lack of credibility. This is achieved by introducing the concept of 'learning', a concept that will be specified in the next sub-section.

#### 4.2.2 Learning

This section will first specify what is meant by learning. This will be followed by a discussion of the manner in which learning is modelled in this thesis. This will be achieved, by introducing a way to capture the fact that the probability mass which characterizes agent's subjective expectations is changing and moving through time onto the central's bank announced money supply path.

Imperfect credibility obviously involves a certain degree of lack of credibility, which is captured by the probability mass characterizing agents' subjective expectations, denoted  $(\rho_t)$ . Moreover, the probability mass characterizing agents' subjective expectations  $(\rho_t)$  is constructed to be dependent on time, shifting from one period to another onto the central's bank announced money supply path. This shifting process which takes place over time occurs due a '*learning* process'.

The learning process is based on two main assumptions:

1. The announcement of the monetary authority becomes more credible (the credibility is increasing) and consequently, this affects the central bank announcement of the money supply path which also becomes more credible.
2. The private sector's attitude towards the announcements of the government is changing over time, from one period of time to the next. This second assumption implies that the private sector is capable of *learning* over time that the

government's announcement is serious and consequently it gives more credit to it.

The idea of learning is modelled by letting  $\rho_t$  (employed as a proxy for learning) change over time, running from an initial  $\rho_0$ , a measure of the initial level of credibility, towards zero and eventually stopping there. This is achieved by imposing/assuming an exogenous learning process (the path followed by  $\rho_t$ ) of the private sector realized in various ways that will be discussed in the next section of this chapter as types of learning.

At moment  $t = 0$  when the disinflation policy is announced, the private sector has a certain degree of credibility,  $\rho$ , in the authorities announcement. So, at  $t = 0$  there is implied some initiated level of credibility that the private sector has for the announcement of the government  $\rho_0 = \rho$ .  $\rho_0 \in [0, 1]$ , where  $\rho_0 = 0$  implies absolute credibility of the private sector in the government's announcement and  $\rho_0 = 1$  implies a complete lack of credibility at the beginning of the learning process. The closer  $\rho_0$  is to zero, the higher the initial level of credibility will be and the closer the perfect foresight case will be, as it can be seen from (4.3). Consequently, the further  $\rho_0$  is from zero, the smaller will be the initial level of credibility in the government's announcement.

Within the learning process, based on the two assumptions presented above, starting from a given certain degree of credibility ( $\rho_0 > 0$ ), the probability mass characterizing agent's subjective expectations  $\{\rho_t\}_{t=0}^T$  is changing over time, decreasing to 0. In other words, for the first presented case of expectation formation (4.3), the probability  $\rho_t$  attached to the scepticism of the private sector (the probability at which private sector is expecting money supply to return to the initial steady state) will decrease while increasing the credibility of the government announcement. When  $\rho_t$  arrives at 0, as shown by any of the equations (4.3) or (4.4) the case of perfect foresight is obtained. The period of time through which the private sector arrives at perfect credibility is denoted by  $N$ .

At this point, there are three cases that can be studied regarding the relationship between the lengths of the periods over which both the disinflation and the learning process discussed here can take place:

1. The first case is when the private sector is learning about the credibility of the government's announcement over the entire period of time over which the dis-

inflation process takes place. In this circumstance, the length of the interval of time over which the two processes take place are equal ( $N = T$ ).

2. The second case is encountered when the private sector's learning process ends in perfect credibility within (or during) the period of time over which the disinflation process takes place. This occurs when the period of time over which the learning process takes place is shorter than that over which disinflation is conducted ( $N < T$ ). This is modelled keeping the learning process at perfect credibility, by setting  $\rho_t = 0$ , for any  $t \in [N, T]$ . In other words, after  $\rho_t$  becomes 0, it stays at this level, unchanged for the rest of the time period over which the disinflation process is conducted.
3. The last case is when the learning process still continues after price stability has been attained. This occurs when the length of the interval over which the learning process takes place is longer than the one over which the disinflation is conducted ( $N > T$ ). It is assumed that over the entire period of time  $T$ , the learning process will never allow the private sector to find itself in the case of perfect credibility. So,  $\rho_t \neq 0$ , for any  $t \in [0, T]$ . The overall learning process is now characterized by  $\{\rho_s\}_{s=0}^{T+J}$ , where  $s \in [0, T + J]$ .  $J = N - T \geq 0$ , is an extra interval of time added to the previous  $T$  in which it is assumed that the learning process still takes place and furthermore, will eventually allow the private sector to get to perfect foresight, such as  $\rho_{T+J} = 0$ . So for  $s \in [T, J]$ , the private sector finds itself still under the learning process.

Two obvious further modelling difficulties that arise for this case and need to be analysed are with respect to:

- (a) The time it takes until  $\rho_{T+J} = 0$ , in other words the length of the interval over which the learning process still takes place after price stability has been attained. What is the value of  $J$ ?
- (b) The path of the learning process  $\rho_s$  in the transition between these extremes.

However, in all three of these cases the learning process takes place over a finite period of time over which the agents eventually arrive at perfect foresight. The learning

process modelled in this manner allows the concept of a *temporary* lack of credibility to be captured.

### 4.3 Types of Learning

So far, the focus has been on the instance in which the probability mass characterizing the agent's subjective expectations,  $\rho_t$ , shifts through time onto the government's announced money supply path over various lengths of time. This length of time may be within, equal to or longer than the period over which price stability is achieved. It is important to note that not only can the learning process lie over different lengths of time but also it can happen in three different ways, corresponding to three different types of learning. These types of learning are differentiated by the way in which perfect credibility is achieved, capturing the path of the evolution of the willingness of the private sector to believe the government's announcement about the money supply, after the change in policy. This section deals with the segment of the path of the learning process over which perfect credibility is achieved, more specifically,  $\{\rho_t\}_{t=0}^N$ , knowing that  $\rho_N = 0$  and remains so thereafter. This can be a straight line or a curve, which can be either concave or convex.

#### 4.3.1 Linear Learning

The first option is simply a sequence  $\{\rho_t\}_{t=0}^N$  that converges linearly to zero in the following manner:

$$\rho_t = \rho_{t-1} - \alpha \frac{\rho_0}{N}, \quad t \geq 1 \quad (4.5)$$

where  $\rho_0 = \rho$  is the initial level of credibility,  $N$  is the length of the interval of time over which the private sector achieves perfect foresight and  $\alpha \in [0, 1]$ .  $\alpha$  captures the time it takes agents to completely believe the central bank's announcements, after which they find themselves under perfect foresight.

$$\rho_t = \rho_0 - \alpha \frac{\rho_0}{N} t, \quad t \geq 1. \quad (4.6)$$

In continuous time one would consider the function:  $\rho(t) : R_+ \rightarrow R_+$ , where:

$$\rho(t) = \rho_0 - \alpha \frac{\rho_0}{N} t, \quad t \geq 1. \quad (4.7)$$

The first derivative of (4.7) with respect to time,  $\alpha \frac{\rho_0}{N}$ , gives the (negative) slope of the path of the learning process. This is also the 'speed of the learning process'. In other words, the slope of the line describing the path of the learning process gives the speed of the learning process. Recall that the faster this line gets to zero, the quicker the learning process will be. So, the steeper its slope, the faster the learning process will be. Given that the first derivative of (4.7) does not depend on time, it can be said that in the case of a linear learning, the speed of the learning process is constant.

When  $\alpha = 1$  the case of the highest speed of the learning process (possible for the given initial level of credibility) is met. In this case the population finds itself in the fastest learning process with constant speed, over the entire period of time over which the disinflation process takes place.

When  $\alpha = 0$  it can be seen that formulation (4.5) perfectly nests the case which allows the study of the effects of a disinflation policy free from the consequences of the implementation of a learning process.

$$\rho_t = \rho_{t-1}, \quad t \geq 1.$$

Since the probability mass which characterizes agent's subjective expectations is not changing over time but rather stays at the same level over the entire period of the disinflation process, this means that the learning process does not take place. This represents the case in which the private agents are not adjusting their expectations from one period to another, in other words, they are not learning. Moreover,  $\rho_0 = 1$  successfully represents the case in which the private sector never learns, carrying on their absolute lack of credibility ad infinitum. This has disastrous effects on the output during disinflation, as the output will never return to its initial steady state level. This case has been employed for the study of the behaviour of the output during a disinflation process in this thesis, as will be seen in section (7.4) which discusses the Volker disinflation. However, in the case discussed in this thesis in Chapter 7, it is assumed that the agents do not trust the announcement of the government at all for the first 5 years only. It is also assumed that perfect foresight is achieved in the sixth year.

### 4.3.2 The Case of Non-Linear Learning

So far, the linear type of learning, which characterizes the case in which the private sector is learning at a constant speed over time towards perfect foresight, has been described.

However, there are more plausible characterizations that can be implemented which allow for the speed of learning to change over time, either increasing or decreasing. This all depends on the evolution over time of the private sector's willingness to consider credible the government's announcement regarding the future path of the money supply. The pace of this evolution can take two forms:

1. At the beginning agents are reluctant to believe the government announcement but as time goes, they start to consider it more credible.
2. At the beginning of the announcement the agents are more inclined to believe the government announcement, but as time passes, they become less inclined to believe it. They either become indifferent, assuming that a low inflation does not present a significant threat and therefore treating it as having minimal importance or, they lose their trust in the announcement of the government because they worry that as the slope of the short-run Phillips curve flattens, the monetary authority may be tempted to renege.

The path of the learning process with respect to time in this case will not be the straight line described as in the previous case, but rather it will be either concave or convex. The second derivative of the learning process equation is going to be either positive or negative, given the acceleration during the learning process. Obviously, there are going to be non-linear equations involved. The following functions are useful for capturing such paths of the learning process. Each of these two cases will be discussed in some detail in the next two sections.

### 4.3.3 Concave Learning

The evolution of a concave process is described by the following sequence  $\{\rho_t\}_{t=0}^N$ , where  $N$  is the length of the interval of time over which the learning process takes



place. This evolves according to:

$$\rho_t = k (a^2 - t^2)^{\frac{1}{2}}, \quad (4.8)$$

where  $a, k > 0$ .

This captures the path of a concave learning process which starts from an initial level of credibility and shows how the credibility is changing over time, through learning, ending at zero, in perfect credibility.

For the first moment,  $t = 0$ ,  $\rho_0$  is the initial degree of credibility considered as a starting condition.

From the given starting condition it can be shown that:

$$k = \frac{\rho_0}{a}. \quad (4.9)$$

**Proof.** The evolution of the learning process is given by (4.8), starting with the initial condition for  $t = 0 \Rightarrow \rho_0 = k(a^2)^{\frac{1}{2}}$ . From here results:

$$\rho_0 = ka \Rightarrow k = \frac{\rho_0}{a}.$$

*Q.E.D.*

According to (4.9), the form of the function (4.8) can be rewritten as:

$$\rho_t = \frac{\rho_0}{a} (a^2 - t^2)^{\frac{1}{2}}. \quad (4.10)$$

Moreover, it is shown that  $a = N$ , where  $N$  is the length of the interval of time over which private sector gets to perfect foresight.

**Proof.** Let  $N$  be the length of the interval of time over which the private sector gets to perfect foresight, while  $T$  is the interval of time over which the disinflation process takes place. After the case of perfect foresight is achieved due to the learning process, the disinflation process continues, knowing that  $\rho_N = 0$  and that the case of perfect foresight is characterized by  $\rho_t = 0$ , for any  $t > N$ . Therefore, for any  $t' \geq N$ , under the case of perfect foresight:  $\rho_{t'} = \rho_N = 0$ . Hence, for  $t' = N$ , from (4.10), it follows that:

$$\rho_N = \frac{\rho_0}{a} (a^2 - N^2)^{\frac{1}{2}} = 0$$

and following that

$$(a^2 - N^2)^{\frac{1}{2}} = 0$$

results

$$a = N.$$

So, (4.10) becomes now:

$$\rho_t = \frac{\rho_0}{N} (N^2 - t^2)^{\frac{1}{2}}. \quad (4.11)$$

*Q.E.D.*

Further, (4.11) is written following a simple recursive process:

$$\frac{\rho_t}{k} = \left\{ \left( \frac{\rho_{t-1}}{k} \right)^2 + (1 - 2t) \right\}^{\frac{1}{2}}.$$

For a comprehensive analysis of the process, this is considered under continuous time. The function is defined by:  $\rho : [0, N] \rightarrow R_+$ , where

$$\rho(t) = \frac{\rho_0}{N} (N^2 - t^2)^{\frac{1}{2}}. \quad (4.12)$$

Given  $\rho_0$  and  $N$ , (4.12) plots the path of a concave function (the second derivative with respect to  $t$  is negative). This captures the intuitive idea that agents may be reluctant to update their priors initially, but as time goes by and the monetary authority sticks to its announced money supply, they increasingly come to believe the announced target path. This case shall be referred to as *concave learning*.

#### 4.3.4 Convex Learning

On the other hand, the evolution of a convex process is described by the sequence  $\{\rho_t\}_{t=0}^N$  which follows:

$$\rho_t = (-1)k \left[ a^2 - (t - a)^2 \right]^{\frac{1}{2}} + \rho_0. \quad (4.13)$$

In analysing this convex process the context applied in the case of concave learning is maintained, meaning that parameters  $a$  and  $k$  are kept the same as in the previous subsection. Considering  $a = N$  the length of the interval of time over which the learning

process takes place, the above relation becomes:

$$\rho_t = -\frac{\rho_0}{N} [N^2 - (t - N)^2]^{\frac{1}{2}} + \rho_0, \quad (4.14)$$

which follows the simple recursive process:

$$\frac{\rho_t}{k} = \left\{ \left( \frac{\rho_{t-1} - \rho_0}{k} \right)^2 + [1 - 2(t - a)] \right\}^{\frac{1}{2}} + \frac{\rho_0}{k}.$$

In continuous time, this is defined by:  $\rho : [0, N] \rightarrow R_+$ , where

$$\rho(t) = -\frac{\rho_0}{N} [N^2 - (t - N)^2]^{\frac{1}{2}} + \rho_0. \quad (4.15)$$

Obviously, capturing a convex path, the second derivative of (4.15) with respect to  $t$  is positive. This reflects the learning path of a population who, although happy to accept that the monetary authority dislikes the current relatively high rate of inflation, nevertheless worry that as the slope of the short-run Phillips curve flattens, the monetary authority may be tempted to renege. The importance of the exploitability of the Phillips curve has been recognized by Ball, Mankiw and Romer (1988) and is a crucial argument in the high inflation equilibria in games of the Barro and Gordon (1983) sort. This case shall be referred to as *convex learning*.

#### 4.3.5 Concave and Convex Learning - In a nut shell

So far, the two types of learning, concave and convex, that have been discussed here, have been each described by a different expression, given by (4.11) or (4.14). However, both these expressions can be represented in only one expression. Thus, the two paths of non-linear types of learning are nested in:

$$\rho_t^\delta = (-1)^\delta k(a^2 - (t - \delta a)^2)^{\frac{1}{2}} + \delta \rho_0, \quad (4.16)$$

when for  $\delta = 0$  the case of concave learning case is met and for  $\delta = 1$  the case of convex learning case is met.  $\rho_t^\delta$  is the notation which allows differentiation between the two types of learning processes, where the superscript  $\delta$  specifies the type of learning process. So  $\rho_t^0$  stands for the concave learning process and  $\rho_t^1$  for the convex learning process.

Clearly, it is possible to imagine intermediate cases, such as a truncated bell-shaped path for  $\rho_t$ . This would capture a situation in which agents initially place little weight on the authorities announcements, as in (4.11). However, after some time (characterized by an inflexion in the path of  $\rho_t$ ), agents once again become more sceptical, as in (4.14). These alternate paths are not dealt with in this thesis.

Given the fact that generally, the public does not trust the government's announcement at the beginning of the change in policy, but is willing to become more cooperative over time, the concave type of learning is considered to be most plausible. However, there are still two difficult questions to answer:

1. What is a reasonable value for  $\rho_0$ ? In other words, how credible is the monetary authority's announcement at date zero?
2. At what point  $T$  does  $\rho_{t>T}$  have to become equal to 0, for any  $t$ ? In other words, how long does it take for agents to arrive at perfect foresight?

The answers to the above two questions will be given by testing different values. The modelling of imperfect credibility presented in this chapter clearly needs to be applied to an actual economy. This is however beyond of the scope of the current chapter. Therefore, Chapter 6 will analyse the behaviour of output in an economy like the one described in Chapter 3 where these settings which allow to model a temporary lack of credibility have been incorporated. An examination of the behaviour of output under imperfect credibility for each of the factors determining the learning process will be undertaken. These factors are: the length of the interval of time over which the learning takes place, the initial level of credibility and the type of learning. Moreover, each of these factors will be studied under both expectation updating rules presented in section 4.1.1, (4.3) and (4.4) respectively. The results will be compared with those of Ireland (1997) who studied the behaviour of output during a disinflationary process under perfect foresight. This comparison is undertaken in order to observe the differences between the output path under perfect and imperfect credibility and to identify the factors that are bringing the changes and to quantify them where changes occur. Therefore, the research questions explored in Chapter 6 of this thesis will be:

1. How will output behave in an environment in which there is a temporary lack of credibility?

2. What will be the output effect of each of the factors which determine the learning process?

Furthermore, Chapter 7 will provide an analysis of the optimal speed of disinflation in such an environment under imperfect credibility/temporarily lack of credibility. So far, only Ireland (1997) has studied the optimal speed of disinflation but under perfect foresight. In addition, there will be an attempt to identify which of the factors determining a learning process influence the optimal speed of disinflation and to quantify these effects when they appear.

#### **4.4 Quantifying the Degree of Lack of Credibility of a Learning Process during a Disinflation Process.**

The previous sections of this chapter discussed the way in which imperfect credibility has been modelled in order to be implemented in the model presented in Chapter 3. The learning process employed in this thesis allows for the study of the effects of a monetary policy during a temporary lack of credibility. Therefore, what is meant by 'learning' in this context, together with the types of learning modelled, have been discussed in some detail. The type of learning process is central to this research as it captures the willingness of the private sector to believe the government's announcement regarding the money supply after the change in policy. Moreover, in this thesis, the degree of lack of credibility in the government's announcement during a disinflation process is quantified. This gives a measure of the lack of credibility which allows a comparison to be made among various learning processes and eventually allows a choice to be made. Therefore, in this section what is meant by the degree of lack of credibility and how this can be measured is discussed. Second, the degree of lack of credibility is quantified for various scenarios in which a learning process takes place. This is followed by a comparison of these scenarios.

The level of lack of credibility is dependent on the surrounding circumstances in which the learning process takes place, characterized by the settings for:

1. The initial level of credibility,  $\rho_0$ .
2. The length of the interval of time over which private sector gets to perfect foresight,  $N$ .

3. The speed of the learning process which has been found to be:

- (a) Constant for the case of linear learning, being given by (4.5).
- (b) Variable for the case of non-linear learning, depending on the evolution of the willingness of the private sector to believe the authorities's announcement, given by (4.11) in the case of concave learning and by (4.14) in the case of convex learning.

To be specific, *the degree to which there is lack of credibility (the degree of lack of credibility)* is the level of distrust exhibited by the private sector with respect to a specific scenario in which a disinflation process takes place. In this context, the *degree of lack of credibility* is measured by the area under the line (in the case of linear type of learning) or the curve (in the case of non-linear type of learning) describing the learning process<sup>2</sup>.

Moreover, keeping the same specification for the type of learning process employed (linear, concave or convex) the area under the curve described depends now on only two parameters: the initial level of credibility ( $\rho_0$ ), and the length of time it takes the private sector to achieve perfect foresight ( $N$ ).

Recall, that the private sector achieves perfect foresight in  $N$  periods of time during a process of disinflation of  $T$  units of time, where  $N < T$ , and  $t \in [0, T]$ . For any  $t \in [N, T]$  it is known that  $\rho_t = 0$ . Again, the significant segment of the path of the learning process is the one over which perfect foresight is achieved,  $\{\rho_t\}_{t=0}^N$ . Therefore, the degree of lack of credibility of a given learning process (for example concave), depends on the initial level of credibility and the length of the interval of time over which the private sector is learning. Given a specific learning process  $\rho_t$ , the area under the line/curve describing it and denoted by  $\Gamma_{N, \rho_0}$ , depends on two parameters:

1. The initial level of credibility,  $\rho_0$ .
2. The length of the learning period over which private sector arrives at perfect foresight,  $N$ .

Hence, it can be said that  $(\Gamma_{N, \rho_0})$  is the measure of the degree of lack of credibility of a learning process for an initial level of credibility  $\rho_0$  and  $N$  periods of time over

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<sup>2</sup>In the (time, initial degree of credibility) plan in the first quadrant.

which perfect foresight is achieved and which is defined by:

$$\Gamma_{N,\rho_0} = \int_0^N \rho(t)dt, \quad (4.17)$$

where,  $\rho_t$  is given by the type of learning process employed in the study.

Regardless of the type of learning process involved, two main points emerge:

1. The degree of lack of credibility is inversely related to the initial level of credibility.
2. The degree of lack of credibility is directly related to the length of the learning period over which private sector arrives at perfect foresight.

The degree of the lack of credibility of the private sector during the learning process ( $\Gamma_{N,\rho_0}$ ) and the initial level of credibility ( $\rho_0$ ), for the same length of the learning period over which private sector gets to perfect foresight ( $N$ ), are inversely related. For the same length of time it takes the private sector to arrive at perfect foresight, the higher the initial level of credibility, the weaker the degree of lack of credibility of the learning process will be.

The degree of the lack of credibility during the learning process ( $\Gamma_{N,\rho_0}$ ) and the length of time it takes the private sector to achieve perfect foresight ( $N$ ) for the same initial level of credibility ( $\rho_0$ ), are directly related. For the same initial level of credibility, the shorter the length of the learning period over which private sector achieves perfect foresight, the smaller the degree of lack of credibility (given by the area under the line/curve describing the learning process) will be. Furthermore each of the types of learning presented in section 4.3 will be discussed in some detail in the context of the degree of lack of credibility.

#### 4.4.1 Linear Learning

As mentioned above, the degree of lack of credibility of the government's announcement for the case of linear learning is given by the area under the straight line which illustrates the evolution of the learning process. Applying (4.17) to the linear type of learning process described by (4.7), the level of credibility is given by:

$$\Gamma_{N,\rho_0} = \int_0^N \rho_0 \left(1 - \frac{\alpha}{N}t\right) dt, \quad (4.18)$$

which by applying<sup>3</sup>:

$$\int_0^x \rho_0 \left(1 - \alpha \frac{\rho_0}{x} t\right) dt = \rho_0 x \left(1 + \frac{\alpha}{2}\right)$$

becomes:

$$\Gamma_{N,\rho_0} = \rho_0 N \left(1 + \frac{\alpha}{2}\right).$$

Since in this work the attention is focused on the two types of non-linear learning and specifically on the concave type, the discussion of the linear learning stops here.

#### 4.4.2 Concave Learning

Under this type of learning the degree of lack of credibility is given by the area under the concave curve which describes the learning process (4.12). This depends on  $\rho_0$ , the initial level of credibility, and  $N$ , the length of the learning period over which the private sector arrives at perfect foresight.

Applying (4.17) to the learning process (4.12), results in:

$$\Gamma_{N,\rho_0} = \int_0^N \frac{\rho_0}{N} (N^2 - t^2)^{\frac{1}{2}} dt, \quad (4.19)$$

which becomes:

$$\Gamma_{N,\rho_0} = \int_0^N \frac{\rho_0}{N} (N^2 - t^2)^{\frac{1}{2}} dt = \frac{\rho_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt. \quad (4.20)$$

To calculate this, it is applied<sup>4</sup>:

$$\int_0^x (x^2 - t^2)^{\frac{1}{2}} dt = \frac{x^2 \pi}{4}. \quad (4.21)$$

Eventually:

$$\Gamma_{N,\rho_0} = \frac{\rho_0}{N} \frac{N^2 \pi}{4} = \frac{\rho_0 \pi}{4} N. \quad (4.22)$$

This result will be used for a more in depth analysis which intends to uncover how the elements defining the scenario in which a disinflation process takes place affects the degree of the level of credibility. This will be achieved by exploring two further aspects. First, how the degree of lack of credibility is affected by a change in the initial level of credibility and how this is going to be affected by a change in the length of time over

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<sup>3</sup>For proof see section 4.6.1.A in the Appendix of this chapter.

<sup>4</sup>For proof see section 4.6.2.A in the Appendix of this chapter.



which the learning process takes place. Second, how the degree of lack of credibility is affected by a  $x\%$  change in the initial level of credibility and how this is going to be affected by a  $x\%$  change in the length of time over which the learning process takes place. Also, the new length of the learning period over which private sector arrives at perfect foresight in order to achieve the same result needs to be identified.

### The Degree of Lack of Credibility - Change Effect

In this section formulas are provided which account for first, how the degree of lack of credibility of the learning process and the length of the learning period over which private sector arrives at perfect foresight, are related and second, how the degree of lack of credibility of the learning process and the initial level of credibility are related. Discussed in detail, these are:

1. What is the change in the degree of lack of credibility of the learning process if the length of the learning period over which the private sector arrives at perfect foresight decreases from  $N'$  to  $N$ , where  $N < N'$ , (in other words, if the private sector learns faster) ?

$$\Delta\Gamma_{N',N} = \frac{\rho_0\pi}{4} (N' - N).$$

**Proof.**

$$\Gamma_{N',\rho_0} = \int_0^{N'} \frac{\rho_0}{(N')} \left( (N')^2 - t^2 \right)^{\frac{1}{2}} dt = \frac{\rho_0}{N'} \int_0^{N'} \left( (N')^2 - t^2 \right)^{\frac{1}{2}} dt \quad (4.23)$$

and

$$\Gamma_{N,\rho_0} = \int_0^N \frac{\rho_0}{N} (N^2 - t^2)^{\frac{1}{2}} dt = \frac{\rho_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt. \quad (4.24)$$

Subtracting (4.24) from (4.23):

$$\begin{aligned} \Delta\Gamma_{N',N} &= \Gamma_{N',\rho_0} - \Gamma_{N,\rho_0} = \\ &= \rho_0 \left( \frac{1}{N'} \int_0^{N'} \left( (N')^2 - t^2 \right)^{\frac{1}{2}} dt - \frac{1}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt \right). \end{aligned}$$

Following (4.21) one gets that:

$$\Delta\Gamma_{N',N} = \rho_0 \left( \frac{1}{N'} \frac{\pi (N')^2}{4} - \frac{1}{N} \frac{\pi (N)^2}{4} \right),$$

which eventually becomes:

$$\Delta\Gamma_{N',N} = \frac{\rho_0\pi}{4} (N' - N).$$

*Q.E.D.*

2. What is the change in the degree of lack of credibility of the learning process if the initial level of credibility increases from  $\rho'_0$  to  $\rho_0$ , where  $\rho_0 < \rho'_0$ ?

$$\Delta\Gamma_{\rho'_0,\rho_0} = \frac{(\rho'_0 - \rho_0)\pi N}{4}. \quad (4.25)$$

**Proof.** It is known that:

$$\Gamma_{N,\rho'_0} = \int_0^N \frac{\rho'_0}{N} (N^2 - t^2)^{\frac{1}{2}} dt = \frac{\rho'_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt. \quad (4.26)$$

Further subtracting (4.24) from (4.26) and using (4.21) gives that:

$$\begin{aligned} \Delta\Gamma_{\rho'_0,\rho_0} = \Gamma_{N,\rho'_0} - \Gamma_{N,\rho_0} &= \frac{\rho'_0 - \rho_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt \\ &= \frac{\rho'_0 - \rho_0}{N} \frac{N^2\pi}{4} \\ &= \frac{(\rho'_0 - \rho_0)\pi N}{4}. \end{aligned}$$

*Q.E.D.*

### **The Degree of Lack of Credibility - Percentage Change Effect**

Here, as in the previous section, formulas are provided which account first, for how the degree of lack of credibility of the learning process and the length of the learning period over which private sector arrives at perfect foresight are related and second, how the degree of lack of credibility of the learning process and the initial level of credibility are related, when there is a  $x\%$  change in the current degree of lack of credibility.

1. What should be the initial level of credibility ( $\rho'_0$ ) so as to reduce by  $x\%$  the current degree of lack of credibility of a concave learning process characterized

by  $(\rho_0, N)$ .  $(\frac{x}{100}\Gamma_{N,\rho_0} = \Gamma_{N,\rho'_0})$ ?

$$\rho'_0 = \frac{\rho_0 x}{100}.$$

**Proof.** Using equations (4.24) and (4.26), and giving that:

$$\frac{x}{100}\Gamma_{N,\rho_0} = \Gamma_{N,\rho'_0} \quad (4.27)$$

Follows that:

$$\begin{aligned} \frac{x}{100} \frac{\rho_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt &= \frac{\rho'_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt \\ &\Downarrow \\ \rho'_0 &= \frac{\rho_0 x}{100}. \end{aligned}$$

*Q.E.D.*

2. What should be the length of the learning period over which the private sector arrives at perfect foresight ( $N'$ ) employed so as to reduce the current  $(\rho_0, N)$  degree of lack of credibility during the learning process by  $x\%$   $(\frac{x}{100}\Gamma_{N,\rho_0} = \Gamma_{N',\rho_0})$ ?

$$N' = \frac{Nx}{100}. \quad (4.28)$$

**Proof.** Using equations (4.24) and (4.23) given that:

$$\frac{x}{100}\Gamma_{N,\rho_0} = \Gamma_{N',\rho_0} \quad (4.29)$$

results

$$\frac{x}{100} \frac{\rho_0}{N} \int_0^N (N^2 - t^2)^{\frac{1}{2}} dt = \frac{\rho_0}{N'} \int_0^{N'} ((N')^2 - t^2)^{\frac{1}{2}} dt.$$

Using (4.21) gives that:

$$\frac{x}{100} \frac{\rho_0}{N} \frac{N^2 \pi}{4} = \frac{\rho_0}{N'} \frac{(N')^2 \pi}{4},$$

which, eventually, is:

$$N' = \frac{Nx}{100}.$$

*Q.E.D.*

### 4.4.3 Convex Learning

Under this type of learning process, the degree of lack of credibility is given by the area under the convex curve which describes the learning process evolution. This obviously depends on  $\rho_0$ , the initial level of credibility and  $N$ , the length of the learning period over which private sector arrives at perfect foresight. Given these,  $(\Gamma_{N,\rho_0})$  is the measure of the degree to which there is lack of credibility during the learning process.

Applying (4.17) to the learning process (4.15), one gets:

$$\begin{aligned}\Gamma_{N,\rho_0} &= \int_0^N \left[ -\frac{\rho_0}{N}(N^2 - (t - N)^2)^{\frac{1}{2}} + \rho_0 \right] dt \\ &= N\rho_0 \left( 1 - \frac{\pi}{4} \right),\end{aligned}$$

knowing that<sup>5</sup>:

$$\int_0^x \left[ -\frac{\rho_0}{x}(x^2 - t^2)^{\frac{1}{2}} + \rho_0 \right] dt = x\rho_0 \left( 1 - \frac{\pi}{4} \right). \quad (4.30)$$

The set of questions considered for the case of concave learning can be answered in this case as well. However, only the first two questions are dealt with here as the other two questions are similar to those discussed in the previous subsection 4.4.2.

1. What is the change in the degree of lack of credibility of the learning process if the length of the learning period over which the private sector arrives at perfect foresight decreases from  $N'$  to  $N$ , where  $N < N'$ , (in other words, if the private sector learns faster)?

$$\Delta\Gamma_{N',N} = \rho_0 (N' - N) \left( 1 - \frac{\pi}{4} \right). \quad (4.31)$$

**Proof.**

We have:

$$\Gamma_{N,\rho_0} = \int_0^N \left[ -\frac{\rho_0}{N}(N^2 - (t - N)^2)^{\frac{1}{2}} + \rho_0 \right] dt \quad (4.32)$$

and

$$\Gamma_{N',\rho_0} = \int_0^{N'} \left[ -\frac{\rho_0}{N'}(N'^2 - (t - N')^2)^{\frac{1}{2}} + \rho_0 \right] dt \quad (4.33)$$

---

<sup>5</sup>For proof see section 4.6.3.A in the Appendix of this chapter.

Subtracting (4.32) from (4.33) gives:

$$\begin{aligned}\Delta\Gamma_{N',N} &= \Gamma_{N',\rho_0} - \Gamma_{N,\rho_0} \\ &= \int_0^{N'} \left[ -\frac{\rho_0}{N'} \left( (N')^2 - (t - (N'))^2 \right)^{\frac{1}{2}} + \rho_0 \right] dt \\ &\quad - \int_0^N \left[ -\frac{\rho_0}{N} \left( N^2 - (t - N)^2 \right)^{\frac{1}{2}} + \rho_0 \right] dt.\end{aligned}$$

Further, using (4.30) results:

$$\begin{aligned}\Delta\Gamma_{N',N} &= \rho_0 \left( N' \left( 1 - \frac{\pi}{4} \right) - N \left( 1 - \frac{\pi}{4} \right) \right) \\ &= \rho_0 (N' - N) \left( 1 - \frac{\pi}{4} \right).\end{aligned}$$

*Q.E.D.*

2. What is the change in the degree of lack of credibility of the learning process if the initial level of credibility increases from  $\rho'_0$  to  $\rho_0$ ,  $\rho_0 < \rho'_0$ ?

$$\Delta\Gamma_{\rho'_0,\rho_0} = N (\rho'_0 - \rho_0) \left( 1 - \frac{\pi}{4} \right).$$

**Proof.**

We have:

$$\Gamma_{N,\rho'_0} = \int_0^N \left[ -\frac{\rho'_0}{N} (N^2 - (t - N)^2)^{\frac{1}{2}} + \rho'_0 \right] dt \quad (4.34)$$

Subtracting (4.24) from (4.33) and using then (4.30) one gets:

$$\begin{aligned}\Delta\Gamma_{\rho'_0,\rho_0} = \Gamma_{N,\rho'_0} - \Gamma_{N,\rho_0} &= N\rho'_0 \left( \frac{1-\pi}{4} \right) - N\rho_0 \left( \frac{1-\pi}{4} \right) \\ &= N (\rho'_0 - \rho_0) \left( \frac{1-\pi}{4} \right).\end{aligned}$$

*Q.E.D.*

#### 4.4.4 An Analysis of Concave and Convex Learning

Since the two non-linear types of learning are embodied in only one mathematical expression, the two expressions giving the degree of lack of credibility for each of the

two types of learning processes, can also be embodied in only one expression. Recall that the two types of learning were both represented by:

$$\rho_t^\delta = (-1)^\delta k(a^2 - (t - \delta a)^2)^{\frac{1}{2}} + \delta \rho_0, \quad (4.35)$$

keeping the same notation for  $\rho_t^\delta$  as in section 4.3.5. Correspondingly, the degree of lack of credibility is represented by:

$$\Gamma_{x,\rho_0}^\delta = \int_0^x \rho_t^\delta dt = \delta x \rho_0 + (-1)^\delta \frac{x \rho_0 \pi}{4}, \quad (4.36)$$

when for  $\delta = 0$  the case of concave learning case is met, whereas for  $\delta = 1$  the case of convex learning is met. Due to the choice of the parameter  $\delta$  which can be either 0 or 1 depending on the case chosen, further notations are introduced to differentiate between the degree of lack of credibility of the learning process under the concave learning case (obtained for  $\delta = 0$ ) as  $(\Gamma_{n,\rho_0}^0)$  and the degree of lack of credibility the convex learning case (obtained for  $\delta = 1$ ) as  $(\Gamma_{n,\rho_0}^1)$ .

Since the linear type of learning is less likely to occur, this section focuses on a comparison of the two types of non-linear learning processes, which are more plausible. For the same initial level of credibility and the same length of the learning period over which perfect foresight is achieved, the degree of lack of credibility of a type of learning process depends on the learning process involved. It is obvious that the degree of lack of credibility, measured for the same initial level of credibility and the same length of the learning period over which perfect foresight is achieved, is smaller under a convex learning process than the one that occurs under the concave case  $(\Gamma_{n,\rho_0}^1 < \Gamma_{n,\rho_0}^0)$ . This section sheds light on the effect of the type of learning on the degree of lack of credibility. Here three issues are to be considered:

- How great is the difference between the degree of lack of credibility that occurs under the two types of non-linear learning processes?
- Keeping constant the initial level of credibility, what should be the value of the length of the learning period over which the private sector achieves perfect foresight so as to maintain the same degree of lack of credibility, when changing from convex to concave type of learning?
- Keeping constant the length of the learning period over which the private sector

achieves perfect foresight, what should be the value of the initial level of credibility so as to maintain the same degree of lack of credibility when changing from convex to concave type of learning?

Analysing the differences between the two types of non-linear learning formulas which account for these relations are provided. The first aspect discussed is:

1. What is the difference between the degree of lack of credibility of a learning process that takes place under concave learning and then under convex learning ( $\Delta\Gamma_{n,\rho_0}^{0,1}$ )?

$$\Delta\Gamma_{N,\rho_0}^{0,1} = N\rho_0 \left( \frac{\pi}{2} - 1 \right).$$

**Proof.**

$$\Delta\Gamma_{N,\rho_0}^{0,1} = \Gamma_{N,\rho_0}^0 - \Gamma_{N,\rho_0}^1 = \frac{N\rho_0\pi}{4} - \left( N\rho_0 - \frac{N\rho_0\pi}{4} \right) = N\rho_0 \left( \frac{\pi}{2} - 1 \right).$$

*Q.E.D.*

It is obvious that the speed of learning involved in the way the private sector's expectations are formed plays a very important role in the evaluation of the degree of level of credibility for the entire disinflation process. For the same initial level of credibility and the same length of time over which learning takes place, the degree of the level of credibility varies with the type of learning. If the speed at which the learning takes place is decreasing, the degree of level of lack of credibility is smaller in comparison to the case in which the speed is increasing. This is explained by the shape of the curves. The conclusion that can be drawn from here is that the beginning of the period when the change in the policy is announced, is crucial for the degree of lack of credibility of the private sector during the learning process. If the agents are reluctant to believe the monetary authority's announcement at the beginning of the change in policy, even if they are becoming more willing to believe the announcements over time (increasing speed), the overall level of lack of credibility is higher than if they are more willing to believe the announcements at the beginning and then become more reluctant as time goes by (decreasing speed). Therefore, the speed of learning at

the beginning of the process is very important. We shall see concrete examples of this when we simulate the effect on output of an incredible disinflation.

The second and the third aspects discussed here take the comparison further to the relationship between the learning period over which the private sector achieves perfect foresight and the initial level of credibility while maintaining the same degree of lack of credibility when changing from a convex to concave type of learning. This is an attempt to gauge to what extent, by keeping unchanged the length of the learning period over which private sector arrives at perfect foresight, the initial level of credibility would need to change so as to maintain the same degree of lack of credibility, when changing from convex to concave learning. This analysis attempts to provide some guidance to the monetary authority on how they should act when the private sector is reluctant to believe their announcement at the beginning of the policy change. Therefore, the next aspect discussed is:

2. What should be the new initial level of credibility ( $\rho_0$ ) if, for the same length of the learning period over which perfect foresight is achieved ( $N$ ), the monetary authority wants to maintain the same effect of the degree of lack of credibility of the learning process under the concave learning case ( $\Gamma_{N,\rho_0}^0$ ) as under the convex learning case ( $\Gamma_{N,\rho'_0}^1$ ). It is given that  $\rho_0 < \rho'_0$ .

$$\rho_0 = \rho'_0 \left( \frac{4}{\pi} - 1 \right).$$

**Proof.**

$$\Gamma_{N,\rho_0}^0 = \Gamma_{N,\rho'_0}^1$$

$$\Downarrow$$

$$\Leftrightarrow \frac{N\rho_0\pi}{4} = N\rho'_0 \left( 1 - \frac{\pi}{4} \right) \Leftrightarrow \rho_0 = \frac{4\rho'_0 \left( 1 - \frac{\pi}{4} \right)}{\pi} \Leftrightarrow \rho_0 = \rho'_0 \left( \frac{4}{\pi} - 1 \right).$$

*Q.E.D.*

For the same degree of lack of credibility that occurs under both non-linear types of learning and the same length of time over which perfect foresight is



achieved ( $N$ ), the initial level of credibility is obviously lower under the convex learning than under concave learning. This shows that even if the initial level of credibility is high, if the private sector is reluctant to believe the monetary authority's announcement at the beginning of the change in policy, the degree of lack of credibility overall is higher than for the case in which the initial level of credibility is lower but the learning process involved in the formation of the expectation of the private sector is faster at the beginning (although decreasing over time). To achieve the same degree of lack of credibility under the concave learning as under the convex learning while maintaining the same length of the learning period over which perfect foresight is achieved, then  $\rho_0 \simeq 0.27\rho'_0$  which assumes an increase in the initial level of credibility of almost 27%. This means that the authorities have to boost their credibility at the beginning of the process if there is any sign that the agents will be reluctant to believe their announcement at the moment of change in policy.

For a disinflation process to take place with a minimum lack of credibility the authorities have two factors they can try to influence. First, is the initial level of credibility of the private sector prior to the announcement so they could start the disinflation process at a higher credibility level, especially if they are aware of the fact that they will not enjoy much trust at the moment of the announcement. Second, is the learning process involved in the formation of the expectations. The policymakers can influence the learning process if the initial level of credibility at the moment of the announcement of the change in policy is known to be at a reasonable level already. This can be achieved by influencing the value of only one variable,  $\rho_t$ , at each moment in time. The credibility of the private sector may be increased by boosting the information they receive and by being more transparent over the entire period of time over which the process takes place. A discussion of this issue is not, however, the purpose of this chapter.

However, the output effects of the type of learning and of the initial level of credibility will be analysed and discussed in Chapter 6, in sections 6.3.1 and 6.3.4 respectively. Also, the effect of the type of learning on the optimal speed of disinflation is discussed in Chapter 7 in section 7.3.2. The effect of the initial level of credibility on the optimal speed of disinflation is discussed in Chapter 7

in section 7.3.4. Therefore, the next aspect discussed is:

3. What should be the new length of the learning period over which perfect foresight is achieved ( $N$ ), if for the same initial level of credibility ( $\rho_0$ ) the monetary authority wants to maintain the same degree of lack of credibility during the learning process under the concave learning case ( $\Gamma_{N',\rho_0}^1$ ) as under the convex learning case ( $\Gamma_{N,\rho_0}^0$ )? It is given that  $N < N'$ .

$$N = N' \left( \frac{4}{\pi} - 1 \right).$$

**Proof.**

$$\Gamma_{N,\rho_0}^0 = \Gamma_{N',\rho_0}^1$$

$$\Downarrow$$

$$N\rho_0\frac{\pi}{4} = N'\rho_0\left(1 - \frac{\pi}{4}\right) \Leftrightarrow N = N'\left(1 - \frac{\pi}{4}\right)\frac{4}{\pi} \Leftrightarrow N = N'\left(\frac{4}{\pi} - 1\right).$$

*Q.E.D.*

It has been seen that when starting from the same initial level of credibility and having the same degree of lack of credibility, the period of time it takes to arrive at perfect foresight is longer under the case of convex learning. This is due to the decreasing speed at which the learning occurs under convex type of learning. The private sector is more willing to believe the announcement of the monetary authority at the beginning of the process, but they are becoming more reluctant as time passes, and this causes the learning process to take place over a longer period of time.

Nevertheless, the output effects of the length of the learning period over which perfect foresight is achieved will be analysed in some detail in Chapter 6, section 6.3.3. The effect of the length of the learning period over which perfect foresight is achieved on the optimal speed of disinflation is discussed in Chapter 7 in section 7.3.3.

This analysis can be extended to the relationship between the degree of lack of credibility and the initial annual inflation rate, the length of the disinflation period

and the expectation updating rules. This would give more insight into the issue of credibility during a disinflation process. Moreover, future research might investigate the relationship between the degree of lack of credibility and the behaviour of output during a disinflationary process. An interesting aspect to observe would be the relationship between the degree of lack of credibility and the output loss during a disinflationary process.

## 4.5 Conclusions

After arguing in favour of the implementation of imperfect credibility, this chapter presented the manner in which this has been realized so it can be incorporated into the New Keynesian model with sticky prices presented in Chapter 3. The assumption of perfect foresight considered in the work of Ireland (1997), King and Wolman (1999), Khan, King and Wolman (2003), are suitable when one wishes to calculate the optimal inflation rate in, or in the neighbourhood of, an unchanging steady state. This assumption is less appealing when one wishes to characterize the path of output in a transition to price stability, particularly for a high initial inflation rate. Moreover, an inappropriate monetary policy (disinflation too quickly, perhaps) may end up being conducted if the policymakers fail to recognize that their policies may not enjoy complete credibility.

Essentially, in this thesis, it has been assumed that agents perceive two possible outcomes for the money supply during a disinflation period: the money supply announced by the central bank and a reversion to an alternative, more inflationary path for the money supply. The expectation of the money supply has been calculated as a weighted mean of these two possible outcomes, where the weight is the probability mass characterizing agent's subjective expectations. Moreover, this weight has been allowed to change over time, going from 1, which is considered to be the case of absolute lack of trust of the private sector in the announcement of the government, to 0, where perfect foresight is attained.

This probability mass characterizing agent's subjective expectations has been employed as a proxy for learning, since this is how the fact that agents are learning has been captured. This probability mass is moving from 1 to 0, shifting through time onto the government's announced money supply path. The path it follows describes

what has been here called a 'learning process'. Obviously, the learning process can be studied starting from any intermediate value of the probability mass characterizing agent's subjective expectations, between 1 and 0, called the initial level of lack of credibility. Three different exogenous types of learning have been employed, each of them capturing the willingness of the private sector to believe the government's announcement about the money supply over time. These are linear and non-linear types of learning, concave and convex respectively. The focus of this chapter is on the two previously mentioned non-linear types of learning, largely presented here together with a comparison between them, since they seem to be more plausible than the linear one.

Alternate paths for the non-linear learning, which have been left on one side in this thesis, can be further considered for further research. As already mentioned in this chapter, an obvious alternate path is a truncated bell-shaped path for  $\rho_t$  which would capture a situation in which agents initially place little weight on the authorities announcements and after some time (characterized by an inflexion in the path of  $\rho_t$ ) they once again become more sceptical.

This chapter also tried to quantify the degree of lack of credibility in the government's announcement during a disinflation process of a specific scenario in which a learning process takes place. Obviously, for the same length of the learning period over which perfect foresight is achieved, starting from the same initial level of lack of credibility, the measure of the degree of lack of credibility during the learning process is higher under a concave type of learning than under a convex one. Starting from such an observation, how the degree of lack of credibility is affected by a change in the initial level of credibility and how this is going to be affected by a change in the length of time over which the learning process takes place was discussed. Second, in order to change by a given percentage, the current degree of lack of credibility of a given learning process, a new level for the initial level of credibility has been identified and also the new length of the learning period over which private sector arrives at perfect foresight in order to achieve the same result. A comparison between the degree of lack of credibility for the case of concave learning and convex learning has been undertaken. For future research, the relationship between the degree of lack of credibility and the behaviour of output during a disinflationary process might investigate along with the relationship between the degree of lack of credibility and the output

loss during a disinflationary process.

Only one more issue must be clarified before turning to explore the effects of a temporary lack of credibility on output and the optimal speed of disinflation during the transition to price stability which is the main aim of this thesis. This is the natural step to follow at this stage and consists of the methodology employed in solving the model in which imperfect credibility has been incorporated. This will be dealt with in some detail in the next chapter.

## 4.6 Appendix

### 4.6.1 A. Linear Learning

In this appendix it is shown that:

$$\int_0^x \rho_0 \left(1 - \alpha \frac{\rho_0}{x} t\right) dt = \rho_0 x \left(1 + \frac{\alpha}{2}\right).$$

**Proof.**

$$\begin{aligned} \int_0^x \rho_0 \left(1 - \alpha \frac{\rho_0}{x} t\right) dt &= \int_0^x \rho_0 dt - \int_0^x \alpha \frac{\rho_0}{x} t dt \\ &= \rho_0 t \Big|_0^x - \alpha \frac{\rho_0}{x} \left(\frac{t^2}{2}\right) \Big|_0^x = \rho_0 x + \alpha \frac{\rho_0}{x} \frac{x^2}{2} \\ &= \rho_0 x \left(1 + \frac{\alpha}{2}\right). \end{aligned}$$

*Q.E.D.*

### 4.6.2 A. Concave Learning

In this appendix it is shown that:

$$\int_0^x (x^2 - t^2)^{\frac{1}{2}} dt = \frac{x^2 \pi}{4}.$$

**Proof.**

$$\int_0^x \sqrt{x^2 - t^2} dt = \int_0^x \frac{x^2 - t^2}{\sqrt{x^2 - t^2}} dt = \int_0^x \frac{x^2}{\sqrt{x^2 - t^2}} dt - \int_0^x \frac{t^2}{\sqrt{x^2 - t^2}} dt,$$

which further is written:

$$\int_0^x \sqrt{x^2 - t^2} dt = x^2 \int_0^x \frac{1}{\sqrt{x^2 - t^2}} dt + \int_0^x \frac{t(-t)}{\sqrt{x^2 - t^2}} dt. \quad (4.37)$$

Using formula:

$$\int_0^x \frac{1}{\sqrt{x^2 - t^2}} dt = \arcsin \frac{t}{x} \Big|_0^x \quad (4.38)$$

we calculate the first term of the expression (4.37):

$$x^2 \int_0^x \frac{1}{\sqrt{x^2 - t^2}} dt = x^2 \arcsin \frac{t}{x} \Big|_0^x. \quad (4.39)$$

To calculate the second term of the expression (4.37) the following formula is applied:

$$\int f'(t)g(t)dt = f(t)g(t) - \int f(t)g'(t)dt, \quad (4.40)$$

where in this case:

$$\begin{aligned} f(t) &= \sqrt{x^2 - t^2}, \\ g(t) &= t. \end{aligned}$$

So, the second term of the sum becomes:

$$\int_0^x \frac{t(-t)}{\sqrt{x^2 - t^2}} dt = t\sqrt{x^2 - t^2} - \int_0^x \sqrt{x^2 - t^2} dt. \quad (4.41)$$

From (4.39) and (4.41), expression (4.37) can now be rewritten as:

$$\int_0^x \sqrt{x^2 - t^2} dt = x^2 \arcsin \frac{t}{x} \Big|_0^x + t\sqrt{x^2 - t^2} - \int_0^x \sqrt{x^2 - t^2} dt$$

↓

$$2 \int_0^x \sqrt{x^2 - t^2} dt = x^2 \arcsin \frac{t}{x} \Big|_0^x + t\sqrt{x^2 - t^2} \Big|_0^x. \quad (4.42)$$

Applying

$$\int_a^b f(z)dz = F(z) \Big|_a^b = F(b) - F(a), \quad (4.43)$$

the first term of the sum becomes:

$$x^2 \arcsin \frac{t}{x} \Big|_0^x = x^2 \left( \arcsin \frac{x}{x} - \arcsin \frac{0}{x} \right) \quad (4.44)$$

$$= x^2 (\arcsin 1 - \arcsin 0) = x^2 \frac{\pi}{2}, \quad (4.45)$$

knowing that:

$$\arcsin 1 = \frac{\pi}{2},$$

$$\arcsin 0 = 0$$

and the second term becomes:

$$t \sqrt{x^2 - t^2} \Big|_0^x = x \sqrt{x^2 - x^2} - 0 \sqrt{x^2 - 0^2} = 0. \quad (4.46)$$

From (4.44) and (4.46), (4.42) is now:

$$2 \int_0^x \sqrt{x^2 - t^2} dt = x^2 \frac{\pi}{2}.$$

Hence:

$$\int_0^x \sqrt{x^2 - t^2} dt = \frac{x^2 \pi}{4}. \quad (4.47)$$

*Q.E.D.*

### 4.6.3 A. Convex Learning

In this appendix it is shown that:

$$\int_0^x \left[ -\frac{\rho_0}{x} (x^2 - (t-x)^2)^{\frac{1}{2}} + \rho_0 \right] dt = x \rho_0 \left( 1 - \frac{\pi}{4} \right). \quad (4.48)$$

**Proof.**

$$\int_0^x \left[ -\frac{\rho_0}{x} (x^2 - (t-x)^2)^{\frac{1}{2}} + \rho_0 \right] dt = -\frac{\rho_0}{x} \int_0^x \sqrt{x^2 - (t-x)^2} dt + \rho_0 \Big|_0^x,$$

which further, using (4.43) is written:

$$\int_0^x \left[ -\frac{\rho_0}{x} (x^2 - (t-x)^2)^{\frac{1}{2}} + \rho_0 \right] dt = -\frac{\rho_0}{x} \mathfrak{S} + x \rho_0, \quad (4.49)$$

where

$$\mathfrak{S} \stackrel{\text{denote}}{=} \int_0^x \sqrt{x^2 - (t-x)^2} dt. \quad (4.50)$$

Further, (4.50) is developed as follows:

$$\begin{aligned} \mathfrak{S} &= \int_0^x \sqrt{x^2 - t^2 + 2xt - x^2} dt \\ &= \int_0^x \sqrt{2xt - t^2} dt \\ &= \int_0^x \frac{2xt - t^2}{\sqrt{2xt - t^2}} dt. \end{aligned}$$

Expanding, this becomes:

$$\mathfrak{S} = \int_0^x \frac{t(x-t)}{\sqrt{2xt-t^2}} dt + \int_0^x \frac{xt}{\sqrt{2xt-t^2}} dt. \quad (4.51)$$

Moreover, applying a further expansion of the second term of the sum,  $\mathfrak{S}$  is written as:

$$\mathfrak{S} = \int_0^x \frac{t(x-t)}{\sqrt{2xt-t^2}} dt - x \int_0^x \frac{x-t}{\sqrt{2xt-t^2}} dt + x^2 \int_0^x \frac{1}{\sqrt{2xt-t^2}} dt. \quad (4.52)$$

The first integral of this expression is going to be solved through parts integration, applying (4.40), where:

$$\begin{aligned} f(t) &= \sqrt{2xt-t^2}, \\ g(t) &= t, \end{aligned}$$

Hence,

$$\int_0^x \frac{t(x-t)}{\sqrt{2xt-t^2}} dt = t\sqrt{2xt-t^2} \Big|_0^x - \int_0^x \sqrt{2xt-t^2} dt.$$

Recalling (4.43) and (4.50), the last relation becomes:

$$\int_0^x \frac{t(x-t)}{\sqrt{2xt-t^2}} dt = x\sqrt{2x^2-x^2} - 0\sqrt{2x0-0^2} - \mathfrak{S} = x^2 - \mathfrak{S}. \quad (4.53)$$

The second term of the expression (4.52) becomes through basic integration:

$$x \int_0^x \frac{x-t}{\sqrt{2xt-t^2}} dt = x(2xt-t^2)^{\frac{1}{2}} \Big|_0^x.$$

Applying (4.43) it becomes:

$$x \int_0^x \frac{x-t}{\sqrt{2xt-t^2}} dt = x(2x^2-x^2)^{\frac{1}{2}} - x(2x0-0^2)^{\frac{1}{2}} = x^2. \quad (4.54)$$



The last term of the expression (4.52), using (4.38) is:

$$\begin{aligned} x^2 \int_0^x \frac{1}{\sqrt{2xt - t^2}} dt &= x^2 \arcsin \left( \frac{t-x}{x} \right) \Big|_0^x \\ &= x^2 \left( \arcsin \frac{x-x}{x} - \arcsin \frac{0-x}{x} \right), \end{aligned}$$

which further becomes:

$$x^2 \int_0^x \frac{1}{\sqrt{2xt - t^2}} dt = x^2 (\arcsin 0 - \arcsin(-1)) = x^2 \frac{\pi}{2}, \quad (4.55)$$

knowing that:

$$\begin{aligned} \arcsin(-1) &= -\frac{\pi}{2}, \\ \arcsin 0 &= 0. \end{aligned}$$

Substituting (4.53), (4.54), (4.55) back in (4.52) one obtains:

$$\mathfrak{S} = x^2 - \mathfrak{S} - x^2 + x^2 \frac{\pi}{2},$$

which is:

$$2\mathfrak{S} = x^2 \frac{\pi}{2}.$$

Recalling the initial notation (4.50),

$$\int_0^x \sqrt{x^2 - (t-x)^2} dt = x^2 \frac{\pi}{4}$$

Substituting this result in the initial expression (4.49) one gets:

$$\int_0^x \left[ -\frac{\rho_0}{x} \left( x^2 - (t-x)^2 \right)^{\frac{1}{2}} + \rho_0 \right] dt = -x\rho_0 \frac{\pi}{4} + x\rho_0 = x\rho_0 \left( 1 - \frac{\pi}{4} \right).$$

*Q.E.D.*

# Chapter 5

## Methodology

### 5.1 Introduction

Chapter 3 presents the model chosen to address the issue of imperfect credibility in a transition to price stability. The New Keynesian model with sticky prices, was employed by Ireland (1997), King and Wolman (1999), Khan, King and Wolman (2003). However, their work only considers the model under the case of perfect credibility. Chapter 4 argues for the case of imperfect credibility and presents the manner in which this has been modelled. The current chapter presents the methodology employed to solve the model of Chapter 3 taking into account the adjustment of imperfect credibility discussed in Chapter 4.

In order to solve the model under the case of imperfect credibility, an expectation updating rule needs to be implemented in conjunction with a learning process (both as described in Chapter 4). However, the addition of these factors makes the models considerably more difficult to solve. The increased difficulty springs from the fact that in this case, the covariance term that might arise from the expectation expression also has to be considered. Moreover, considerably more attention has been given to solving linear rational expectation models as opposed to non-linear stochastic models. Alternative approaches have been described by Taylor and Uhlig (1990), Marimon and Scott (1999). Judd (1998) also discusses a number of available solution methods, however, it is clear that there is a great deal of work still to be done in the area of non-linear solution methods under uncertainty. This chapter presents the way the model of this thesis has been solved. Essentially, this is realised by mapping a non-linear stochastic problem into a problem that can be numerically solved.

The outline of the chapter is as follows: Section 5.2 presents a general modelling

framework in which is pin-pointed a more specific framework, representative for the model discussed in this thesis. Section 5.3 presents the way this particular model has been solved. Section 5.4 describes how the model of Chapter 3 in which was incorporated the imperfect credibility of Chapter 4 have been dealt with.

## 5.2 The Modelling Framework

This section presents a general modelling framework which pin-points a more specific framework in which the model described in Chapter 3, adjusted with the imperfect credibility of Chapter 4, also falls. The main component relations of such a framework are an equilibrium condition, an expectation variable and an expectation updating rule.

### 5.2.1 The General Case

This section describes the general case of a class of dynamic stochastic models, considered in an economic system, in which arbitrage-free equilibria are enforced through the collective, decentralized actions of atomistic dynamically optimizing agents. It is assumed that agents are rational in the sense that their expectations are consistent with the implications of the model as a whole. Attention is limited to stochastic dynamic models in which at the beginning of period  $t$ , an economic system emerges in a state  $s$ . Agents observe the state of the system and, by pursuing their individual objectives, produce a systematic response  $x_t$  governed by an *equilibrium condition* which in the structural form, is written:

$$f(E_t[x_{t+1}], x_t, x_{t-1}, \dots, x_{t-p}, E_t[s_{t+1}], s_t, s_{t-1}, \dots, s_{t-q}, Z_{t+1}; \theta) = 0. \quad (5.1)$$

The equilibrium condition governing the response  $x_t$  depends on expected values of next period's state and response as well as on current and past periods's state and action. In a shorter form, this may be rewritten as:

$$f(E_t[x_{t+1}], \tilde{x}_t, E[s_{t+1}], \tilde{s}_t, Z_{t+1}; \theta) = 0, \quad (5.2)$$

where it was denoted

$$\tilde{x}_t = \{x_t, x_{t-1}, \dots, x_{t-p}\}$$

and

$$\tilde{s}_t = \{s_t, s_{t-1}, \dots, s_{t-q}\},$$

where  $f : R^{\underbrace{d_x + d_x + \dots + d_x}_{p+2}} \times R^{\underbrace{d_s + d_s + \dots + d_s}_{q+2}} \times R^{d_x} \times R^{d_\theta} \rightarrow R^{d_x}$  is the equilibrium (valued) function.  $d_x$  is the dimensionality of the response space  $X \subseteq R^{d_x}$  which contains the admissible responses of the model economy given by the number of elements of the vectors  $E[x_{t+1}], x_t, x_{t-1}, \dots, x_{t-p} \in R^{d_x}$  which represent the response (endogenous) variables, indexed after  $i = \overline{1, n}$ .  $d_s$  is the dimensionality of the state space  $S \subseteq R^{d_s}$  where  $E[s_{t+1}], s_t, s_{t-1}, \dots, s_{t-q}$ , belonging to  $R^{d_s}$  are the state (control) variables.  $S$  and  $X$  are both closed convex nonempty sets.  $d_z$  is the dimension of the expectation space, where  $Z_{t+1} \in R^{d_x}$  is the expectation variable, and  $\theta \in R^{d_\theta}$  is the vector of parameters of the model.

The expectation variable  $Z_{t+1}$  is given by:

$$Z_{t+1} = E_t [h(x_{t+1}, x_t, x_{t-1}, \dots, x_{t-p'}, s_{t+1}, s_t, s_{t-1}, \dots, s_{t-q'})], \quad (5.3)$$

where the expectation function  $h : R^{\underbrace{d_x + d_x + \dots + d_x}_{p'+2}} \times R^{\underbrace{d_s + d_s + \dots + d_s}_{q'+2}} \rightarrow R^{d_x}$

The expectation up-dating rule in the general form is:

$$E_t [s_{t+1}] = E_t [z(s_{t+1}, s_t, s_{t-1}, \dots, s_{t-q''}, x_{t+1}, x_t, x_{t-1}, \dots, x_{t-p''}, \epsilon_{t+1})], \quad (5.4)$$

where  $z : R^{\underbrace{d_s + d_s + \dots + d_s}_{p''+2}} \times R^{\underbrace{d_x + d_x + \dots + d_x}_{q''+2}} \rightarrow R^{d_s}$  is the state transition function which gives the evolution of the state variable and the exogenous random shocks  $\epsilon_t$  are identically distributed over time, mutually independent, and independent of past states and responses.

The models differ from one another in: (1) the number of lags of both state and response variables of each of the function presented may have, thus in the various values of the parameters  $p, q, p', q'$  and  $p'', q''$  that appears in the main relations stated so far: (a) equilibrium function (5.1), (b) expectation function in (5.3), (c) state transition function in (5.4) and also in (2) the form of the valued, expectation and transition functions.

Before moving on to present the solution method employed, a more specific and simple model will be presented in the next subsection, as this closely matches the

model for which this chapter is intended to provide a solution.

## 5.2.2 The Particular Model

Given the purpose of this research, attention needs to be focused on a specific type of model characterised by values of the parameters  $p$ ,  $q$ ,  $p'$ ,  $q'$  and  $p''$ ,  $q''$  in the main relations (5.1), (5.3), (5.4). These will be discussed in some detail further.

### 1. The equilibrium condition

The equilibrium condition varies, depending in (5.1) on  $p$  and  $q$ , the number of lags of state and response variables in the model. Governing the response,  $x_t$ , might depend on past, current and next period's values of state and response variable. Presented here is a stochastic model in which agents observe the state of the system and, by pursuing their objectives, produce a systematic response  $x_t$  governed by an equilibrium condition that depends on expectations of the following period's state and response as well as on their current values.

$$f(E_t[x_{t+1}], x_t, E_t[s_{t+1}], s_t, Z_{t+1}; \theta) = 0,$$

where  $f : R^{d_x} \times R^{d_x} \times R^{d_s} \times R^{d_s} \times R^{d_z} \rightarrow R^{d_x}$ <sup>1</sup>.

Miranda and Fackler (2002) discuss a stochastic model in which agents observing the state of the system produce a systematic response  $x_t$  governed by an equilibrium condition that depends only on expectations of the following period's state and action.

$$f(E_t[x_{t+1}], E_t[s_{t+1}], Z_{t+1}; \theta) = 0,$$

where  $f : R^{d_x} \times R^{d_s} \times R^{d_z} \rightarrow R^{d_x}$ .

Moreover, there are various types of expectation functions and expectation updating rules.

### 2. The expectation function

The expectation functions differ across models in the number of lags of state and response variables  $p'$ ,  $q'$  in (5.3), given by the specifications of the model. For the purpose

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<sup>1</sup>Due to the fact that  $\theta$  is only the vector of parameters of the model, from now on it will not be mentioned while defining the domains of the functions presented in this chapter.

of this research, attention is focused on models in which the expectation variable is restricted to depend only on the next period's state and action:

$$Z_{t+1} = E_t [h(s_{t+1}, x_{t+1})], \quad (5.5)$$

where  $h$  is the expectation function:  $h : R^{d_s} \times R^{d_x} \rightarrow R^{d_z}$  defined as:

$$h(x_{t+1}, s_{t+1}) = \{h_i(x_{t+1}, s_{t+1})\}_{i=1}^m,$$

where the function  $h_i$  takes the specific form:

$$h_i(x_{t+1}, s_{t+1}) = x_{t+1}^{a(i)} s_{t+1}^{b(i)}, \quad i = \overline{1, m} \quad (5.6)$$

where  $h_i : R^{d_x} \times R^{d_s} \rightarrow R^{d_z}$ , and both  $a(i)$  and  $b(i)$  have the domain given by the set  $\{1, \dots, m\}$  taking values generally in nothing else than a set (group) of the parameters of the model and algebraic operations between them, denoted by  $G$ . So,  $a, b : \{1, \dots, m\} \rightarrow G$ .

### 3. The transition function

Models also vary according to  $p''$  and  $q''$  in (5.4), the number of lags of the state and response variables that appear in the expectation updating rule (or transition function) of the state variable. Along with the variables of the function, an important role is also played by the form of the function  $z$ . In the example that is the focus of this chapter, the transition function is the identity function.

Hence, the expectation updating rule is:

$$E_t[s_{t+1}] = E_t[z(s_{t+1})],$$

where  $z : R^{d_s} \rightarrow R^{d_s}$  is the state transition function. Here, as well as the form of function  $z$ , an important issue is the way the expectation operator is approximated. This will be discussed in more detail in the next section.

Models in which the transition function depends on the future values of the state variable and the current (or a past state) have been met in the literature. As an example, Miranda and Fackler (2002) studied the case in which the expectation updating rule depends only on current values of both state and response variables and no next



period or past values are involved. Their economic system develops to a new state  $s_{t+1}$  dependent on the current state  $s_t$  and the response  $x_t$ . They consider that the new state depends as well on an exogenous random shock  $\epsilon_{t+1}$ , which is realized only after the system responds at time  $t$ :

$$E_t[s_{t+1}] = E_t[z(s_t, x_t, \epsilon_{t+1})],$$

where the state transition  $z : R^{d_s} \times R^{d_x} \rightarrow R^{d_s}$ .

The specification that the response depends only on the expectation of the subsequent period's state and response in any period brings additional implications which do not at first appear. By implementing new accounting variables, responses can be made reliant on expectations of states and responses further in the future.

So far, the general framework and the particular model have been presented. Next the way the particular model has been approached will be presented.

### 5.3 Solving the Model

While linear and some non-linear models can be solved using several numerical methods, nonlinear stochastic models are more difficult to deal with. The presence of expectations in a non-linear model augments its degree of complexity and therefore it receives little attention in the literature. In this case, increased difficulty springs from the fact that the covariance term that might arise from the expectation expression also has to be considered. This section presents the way in which the non-linear dynamic model with expectations specified in the previous section is solved, referring to the particular form.

This is achieved in two steps. The first step entails mapping the non-linear stochastic problem into a form more convenient to work with numerically. The second step then constructs the equilibrium solution to that simpler problem. Given that, for the case of perfect foresight the expectation variables are redundant as are the covariance terms, the solution to the model under perfect foresight requires only the second of these steps.

### 5.3.1 Step 1

The first step entails mapping the non-linear stochastic problem into a form more convenient to work with numerically. In order to achieve this, first the way the expectation operator is approximated is specified and then the expectation updating rule is calculated. Finally, the expectation variable is computed. Central here is the computation of the covariance term as this allows to make the solution of the model more accessible. This step is concluded with the model written in an explicit form.

Generally, *the expectation operator* needs to be approximated. This is done by a discrete distribution that takes on value  $v_j$  with probability  $w_j$  as follows (Judd (1998), Marimon and Scott (1999), Miranda and Fackler (2001)):

$$E[f(v_t)] \simeq \sum_{j=0}^{k+1} w_j f_j(v_{t+1-j}). \quad (5.7)$$

Given this, the expectation updating rule of the state variable and the expectation variable are computed as follows:

1. First, *the expectation updating rule* of the state variable  $E[s_{t+1}]$ :

$$E[s_{t+1}] = E[z(s_{t+1})] = \sum_{j=0}^{k+1} w_j z_j(s_{t+1-j}) = w_0 z_0(s_{t+1}) + w_1 z_1(s_t) + \dots + w_{k+1} z_{k+1}(s_{t-k}),$$

where  $w_0, w_1, \dots, w_k$  are probabilities associated to the distribution function  $z$ .

2. Second, *the expectation variable*  $Z_{t+1}$ :

$$Z_{t+1} = \left\{ E \left[ h_i \left( x_{t+1}^{a(i)}, s_{t+1}^{b(i)} \right) \right] \right\}_{i=1}^m,$$

where we apply

$$E[XY] = E[X] + E[Y] + cov(X, Y).$$

Hence:

$$Z_{t+1} = \left\{ E \left[ x_{t+1}^{a(i)} \right] + E \left[ s_{t+1}^{b(i)} \right] + cov \left( x_{t+1}^{a(i)}, s_{t+1}^{b(i)} \right) \right\}_{i=1}^m.$$

Further, for *the calculation of the covariance term* we use:

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<sup>2</sup>where  $h : R^{d_x} \times R^{d_s} \rightarrow R^{d_z}$ ,  $h(x, s) = xs$ , and both  $a(i)$  and  $b(i)$  have the domain given by a finite set of integers  $I$  taking values generally in nothing else than a set of the parameters of the model and algebraic combination of them denoted by  $S$ ;  $a, b : I \rightarrow S$ .



$$\text{cov}(X, Y) = E[(X - E[X])(Y - E[Y])],$$

which leads to:

$$\text{cov}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}) = E \left[ z \left( \left( x_{t+1}^{a(i)} - E[x_{t+1}^{a(i)}] \right) \left( s_{t+1}^{b(i)} - E[s_{t+1}^{b(i)}] \right) \right) \right].$$

The expectations operator (5.7) is applied again leading to:

$$\text{cov}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}) = \sum_{j=0}^{k+1} w_j z_j \left( \left( x_{t+1-j}^{a(i)} - E[x_{t+1}^{a(i)}] \right) \left( s_{t+1-j}^{b(i)} - E[s_{t+1}^{b(i)}] \right) \right), \quad (5.8)$$

where further we use in the computation of the covariance:

$$E[x_{t+1}^{a(i)}] = \sum_{j=0}^{k+1} w_j z_j (x_{t+1-j}^{a(i)})$$

and

$$E[s_{t+1}^{b(i)}] = \sum_{j=0}^{k+1} w_j z_j (s_{t+1-j}^{b(i)}),$$

where  $w_j$ , are probabilities associated to the distribution function  $z_j : R^n \rightarrow R^n$  which is the identity function, for all  $j = \overline{0, k+1}$ .

Thus,

$$Z_{t+1} = \left\{ E[x_{t+1}^{a(i)}] \right\}_{i=1}^m + \left\{ E[s_{t+1}^{b(i)}] \right\}_{i=1}^m + \left\{ \text{cov}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}) \right\}_{i=1}^m$$

or equivalent:

$$Z_{t+1} = \bar{E}[x_{t+1}^{a(i)}] + \bar{E}[s_{t+1}^{b(i)}] + \overline{\text{cov}}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}),$$

where

$$\bar{E}[x_{t+1}^{a(i)}] = \left\{ E[x_{t+1}^{a(i)}] \right\}_{i=1}^m, \quad \bar{E}[s_{t+1}^{b(i)}] = \left\{ E[s_{t+1}^{b(i)}] \right\}_{i=1}^m$$

and

$$\overline{\text{cov}}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}) = \left\{ \text{cov}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}) \right\}_{i=1}^m.$$

At this stage, the *explicit form of the system* finally, can now be written as follows:

$$E[x_{t+1}] = f \left( E[x_{t+1}], \bar{E}[x_{t+1}^{a(i)}], \tilde{x}_t, E[s_{t+1}], \bar{E}[s_{t+1}^{b(i)}], \tilde{s}_t, \overline{\text{cov}}(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}); \theta \right)$$

or:

$$E[x_{t+1}] = f\left(\tilde{E}[x_{t+1}], \tilde{x}_t, \tilde{E}[s_{t+1}], \tilde{s}_t, \overline{cov}\left(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}\right); \theta\right),$$

where:

$$\tilde{E}[x_{t+1}] = \left\{ E[x_{t+1}], \bar{E}\left[x_{t+1}^{a(i)}\right] \right\}, \tilde{E}[s_{t+1}] = \left\{ E[s_{t+1}], \bar{E}\left[s_{t+1}^{b(i)}\right] \right\}.$$

The model in its explicit form is:

$$E[x_{t+1}] = f\left(E[x_{t+1}], \bar{E}\left[x_{t+1}^{a(i)}\right], \tilde{x}_t, \tilde{E}[s_{t+1}], \tilde{s}_t, \overline{cov}\left(x_{t+1}^{a(i)}, s_{t+1}^{b(i)}\right); \theta\right) \quad (5.9)$$

now, in a more convenient form to work with.

### 5.3.2 Step 2

The second step constructs the equilibrium solution to the simpler problem (5.9). A non-linear model has to be solved using iterative numerical methods, since there is no analytic solution to serve this purpose. For this purpose, two main general solution methods based on different techniques are known: (1) first derivatives of the model and (2) first order methods. Newton's method can be counted in the first category. For the second class of methods the most famous is the Gauss-Seidel method together with the Jacobi solution method and the optimized procedures derived from them such as: Successive Over Relaxation (SOR) draw from Gauss-Seidel method, Jacobi over-relaxation (JOR) or Fast Gauss-Seidel (FGS).

There are several known methods for solving non-linear models with forward expectations based on the iterative procedures mentioned above. (1) The stacked solution method. This method can be applied using any of the nonlinear solution methods already mentioned after writing it as a system of equations (stacking the system of equations for each time period  $t$ ) and ordering them (if the method is order-dependent like Gauss-Seidel method). The stacked Newton method with the Jacobian matrix can also be applied. (2) The Fair-Taylor (1983) solution method proposes a first order solution method for a nonlinear dynamic model with consistent expectations. (3) The shooting method starts from the reduced form of the stacked system. This method rewrites the model solution problem as a two-point boundary value problem where the boundary values are initial and terminal values of the expectations. Last but not

least is (4) the terminal conditions solution method. An excellent presentation of the computational techniques mentioned here can be found in Pauletto (1997). First order condition solution methods for nonlinear rational expectation models are extensively analyzed in Fisher (1992) who also applies these methods to large models of the UK economy.

For the purpose of this research, the method chosen on criteria of efficiency and speed is the *Jacobi over-relaxation* solution method. This is defined by the iteration  $j$

$$x_t^j = f \left( x_t^{j-1}, \tilde{x}_t, \tilde{s}_t; \theta \right), \quad (5.10)$$

where

$$\tilde{x}_t = \{x_{t-1}, \dots, x_{t-p}\}$$

and

$$\tilde{s}_t = \{s_t, s_{t-1}, \dots, s_{t-q}\}$$

and generates a new solution using the previous iteration solution on the right-hand side of each equation. Iteration proceeds until a fixed point is found where:

$$x_t^j = x_t^{j-1}. \quad (5.11)$$

In order to solve the non-linear rational expectation model (5.9), the iterative numerical method of Jacobi presented earlier is applied. Moreover, this method can be easily applied if the number of lags of response and state variables are the same ( $p = q$ ) to allow the calculation of the covariances between  $E[x_{t+1}]$  and  $E[s_{t+1}]$ .

At moment  $t+1$  the only unknown is  $E[x_{t+1}]$  which also enters the expectation variable. Starting from an initial guess  $E[x_{t+1}]^0 = x$ , each iteration  $j$ ,  $j = \overline{1, J}$  generates a new vector of values  $E[x_{t+1}]^j$  based on the previous iteration.

Convergence is achieved when

$$\max^i \left| E[x_{it+1}]^j - E[x_{it+1}]^{j-1} \right| < \varepsilon,$$

where  $i = \overline{1, n}$ ,  $n$  finite, is the index of an element in the vector  $E[x_{t+1}]$  and  $\varepsilon$  is chosen precision.

To find an initial value of  $cov[s_{t+1}, x_{t+1}]$ , the model is initially solved assuming that there is no correlation between the state and response variables  $E[s_{t+1}]$  and

$E[x_{t+1}]$ , first under the assumption  $w_0 = 1$  and  $\sum_{j=1}^{k+1} w_j = 0$  and then assuming that they are fully correlated,  $w_{k+1} = 0$  and  $\sum_{j=0}^k w_j = 1$ . Given the outcomes of these experiments, the starting value of the covariances is constructed according to (5.8), where the expected value of the price  $E[x_{t+1}^{a(i)}]$  is calculated according to (5.7) given that at the first iteration,  $z_0(x_{t+1}^{a(i)}) = x_{t+1}$  obtained under the first assumption and  $z_{k+1}(x_{t+1}^{a(i)}) = x_{t+1}$  under the second one.

After all the settings for the initial point have been done, the iterative process starts and continues until a fixed point is reached. During this process, new covariances are calculated at each new iteration, based on the outcome of the last iteration.

$$E[x_{t+1}]^j = \phi f \left( \tilde{E}[x_{t+1}]^{j-1}, \tilde{x}_t, \tilde{E}[s_{t+1}], s_t, \overline{cov} \left( x_{t+1}^{a(i)}, s_{t+1}^{b(i)} \right)^{j-1}; \theta \right) + (1 - \phi) E[x_{t+1}]^{j-1}.$$

The scalar  $\phi \in (0, 1)$  is a relaxation parameter which reduces the change between the iterations in order to speed up the convergence.

## 5.4 Example

In this section, the steps laid out in the previous section are applied to the model of this thesis in which imperfect credibility was incorporated in order to reach a solution. As seen in Chapter 3, firms are divided into two categories. For  $t = 2l, l \in Z_+$ , firms from the first set can freely change their price for output,  $p_{1,t}(i)$ , while the firms belonging to the second set must sell output at the same price set a period before,  $p_{2,t}(i) = p_{2,t-1}(i)$ , unless they pay the fixed cost  $k > 0$ , measured in terms of labour. For  $t = 2l + 1$ , the roles are reversed and the first set of firms keep prices unchanged,  $p_{1,t}(i) = p_{1,t-1}(i)$  unless they are willing to pay the fixed cost  $k$ , while the second set of firms, can freely set new prices. The model assumes that firms are constantly re-evaluating their pricing strategy and weighting the benefits of holding prices fixed versus the alternative of changing prices and incurring the fixed penalty. However, at moment  $t$  the firms belonging to the set of firms that can freely change price are able to choose between the two strategies, depending on whether the inflation rate is moderate or high. At more moderate rates of inflation or in face of gradual changes in

the monetary stance, they are more likely to keep their prices constant for two periods and hence avoid the cost  $k$  (single price strategy). On the other hand, in the case of a high inflation or in the face of sharp changes of the monetary stance, firms are more likely to choose a new price and pay the cost  $k$  (two price strategy). The criterion for a price-setting decision at time  $t$  is to maximize the returns to the shareholders. In this chapter the focus will be on solving the model for the single price strategy which was discussed in 3.3.1. It has to be mentioned that solving the model for the two price strategy follows exactly the same procedure as for the single price strategy which will be discussed in this section. Therefore, it is unnecessary to present the solution for the two price strategy here as well.

Before the two steps presented in the previous section are followed, however, a third one has been added in order to write the model in a structural form and in terms of the state, response and expectation variables. The first step describes the expectation updating, the expectation variable, the way the covariance term has been calculated and ends with the model written in the explicit form. The second step illustrates the way the iterative solution method described in the earlier section (at Step 2) has been operated. It is seen that  $k = 1$  and  $p = q = 0$ .

### 5.4.1 Step 0

This additional step is necessary in order to write the equations of the model (the price (3.14) and the profit (3.16) equations) in terms of the state, response and expectation variables and to present the model in a structural form, ready for the next step.

Given that:

$$Y_t = \frac{M_t}{P_t},$$

these relations will be rewritten so as to be expressed in terms of the response and state variables,  $P_t$  and  $M_t$ .

First, from equation (3.14), the price is:

$$p_t(i) = \frac{b}{b-1} \frac{P_t^{1-b} M_t^{2b-1} + \beta E_t P_{t+1}^{1-b} M_{t+1}^{2b-1}}{P_t^{\alpha-b} M_t^{2b-\alpha-1} + \beta E_t P_{t+1}^{\alpha-b} M_{t+1}^{2b-\alpha-1}}, \quad (5.12)$$

under the single price strategy, from equation (3.16) and

$$p_t(i) = \frac{b}{b-1} P_t^{b-1} M_t^{1-\alpha}$$

or

$$p_{t+1}(i) = \frac{b}{b-1} P_t^{b-1} M_{t+1}^{1-\alpha}$$

under the two price strategy.

Second, from (3.13), the profit  $\Pi_t(i)$  for the case of the single price strategy in terms of response and state variables is:

$$\begin{aligned} & P_t^{b-1} M_t^{1-b} \left\{ P_t^{-\alpha} M_t^\alpha \left[ \frac{p_t(i)}{M_t} \right]^{1-b} - \left[ \frac{p_t(i)}{M_t} \right]^{-b} \right\} + \\ & + \beta E_t P_{t+1}^{b-\alpha-1} M_{t+1}^{1-b+\alpha} \left[ \frac{p_{t+1}(i)}{M_{t+1}} \right]^{1-b} - E_t P_{t+1}^{b-1} M_{t+1}^{1-b} \left[ \frac{p_{t+1}(i)}{M_{t+1}} \right]^{-b} - \beta \chi_{t+1}(i) k. \end{aligned} \quad (5.13)$$

For each of the possible prices that may be chosen (single or two price strategy), the profit is calculated and through a simple maximization scheme the price that maximizes the profit is identified. In this example, the case in which the profit is maximized under the single price strategy is presented. The (maximizing) price  $p_t(i)$  given by (5.12) is expressed as:

$$p_t(i) = \Psi \left( P_t, M_t, E \left[ P_{t+1}^{1-b} \right], E \left[ M_{t+1}^{2b-1} \right], E \left[ P_{t+1}^{\alpha-b} \right], E \left[ M_{t+1}^{2b-1-\alpha} \right], Z_{t+1}^\Psi \right),$$

where

$$Z_{t+1}^\Psi = \left\{ E \left[ P_{t+1}^{1-b} M_{t+1}^{2b-1} \right], E \left[ P_{t+1}^{\alpha-b} M_{t+1}^{2b-a-1} \right] \right\}$$

and the profit  $\Pi_t(i)$  given by (5.13) is represented by:

$$\Pi_t(i) = \Theta \left( P_t, M_t, E \left[ P_{t+1}^{b-\alpha-1} \right], E \left[ M_{t+1}^{1-b+\alpha} \right], E \left[ P_{t+1}^{b-1} \right], E \left[ M_{t+1}^{1-b} \right], p_t(i), p_{t+1}(i), Z_{t+1}^\Theta \right),$$

where

$$Z_{t+1}^\Theta = \left\{ E \left[ P_{t+1}^{b-\alpha-1} M_{t+1}^{1-b+\alpha} \right], E \left[ P_{t+1}^{b-1} M_{t+1}^{1-b} \right] \right\}.$$

Denoting:

$$\bar{E} [M_{t+1}] = \left\{ M_{t+1}^{2b-1}, M_{t+1}^{2b-a-1}, M_{t+1}^{1-b+\alpha}, M_{t+1}^{1-b} \right\},$$

$$\bar{E} [P_{t+1}] = \left\{ P_{t+1}^{1-b}, P_{t+1}^{\alpha-b}, P_{t+1}^{b-1}, P_{t+1}^{b-\alpha-1} \right\},$$

$$Z_{t+1}^{\Psi\Theta} = \left\{ Z_{t+1}^\Psi, Z_{t+1}^\Theta \right\},$$

the equation of the maximized profit can be given as:

$$\Pi_t(i) = \Theta \left( M_t, P_t, \bar{E} [M_{t+1}], \bar{E} [P_{t+1}], Z_{t+1}^{\Psi\Theta} \right).$$

In the structural form, the problem is written as:

$$f \left( \bar{E} [P_{t+1}], P_t, \bar{E} [M_{t+1}], M_t, Z_{t+1}^{\Psi\Theta}; \theta \right) = 0,$$

where  $f : R^{d_P+d_P} \times R^{d_M+d_M} \times R^{d_z} \times R^{d_\theta} \rightarrow R^{d_P}$  is the equilibrium (valued) function where the dimensionalities of the response, state and expectation spaces are equal:  $d_P = d_M = d_Z = d_\theta = n = 202$  (the number of elements in the vector),  $\theta$  is vector of parameters of the same dimension.

The next step consists of mapping the non-linear stochastic problem into a more convenient form to work with numerically.

### 5.4.2 Step 1

At moment  $t + 1$  the only unknown is  $E [P_{t+1}]$  which enters the expectation variable. However,  $E [M_{t+1}]$ , the expected money supply, is given by (5.7) in which  $k = 1$ . This is calculated as follows:

$$E [M_{t+1}] = E [z(M_{t+1})] = \sum_{j=0}^1 w_j z_j(M_{t+1-j}) = w_0 z_0(M_{t+1}) + w_1 z_1(M_t),$$

where the weights  $w_0$  and  $w_1$  are the probabilities associated by the private sector to each of the two outcomes of  $z$ . Therefore,  $w_0 = (1 - \rho_t)$  expresses the probability with which the private sector is expecting the monetary authority's money supply ( $z_0(M_{t+1}) = M_{t+1}$ ) and  $w_1 = \rho_t$  expresses the probability with which they are expecting an alternative, more inflationary path of the money supply for which two choices have been modelled in this thesis as presented in section 4.2.1. First, agents perceive the authorities as reverting to the previous steady state inflation, case in which they expect the money supply to return to its previous steady state ( $z_1(M_t) = \theta_{-1} M_t$ ),  $\rho_t \in [0, 1]$ . Second, agents fear that the government will 'run out of steam' so that at time  $t$  (for  $0 < t < T$ ) the growth rate of the money stock will be equal to the growth rate between  $t - 1$  and  $t$  ( $z_1(M_t) = \theta_{t-1} M_{t+j-1}$ ),  $\rho_t \in [0, 1]$ . The expected money supply  $E [M_{t+1}]$

is calculated according to (4.3) or (4.4), as a weighted mean between the first outcome and each of the two alternatives of the second one.

The expectation variable  $Z_{t+1}$  is given by:

$$Z_{t+1} = Z_{t+1}^{\Psi\Theta} = \left\{ E \left[ P_{t+1}^{1-b} M_{t+1}^{2b-1} \right], E \left[ P_{t+1}^{\alpha-b} M_{t+1}^{2b-\alpha-1} \right], E \left[ P_{t+1}^{b-\alpha-1} M_{t+1}^{1-b+\alpha} \right], E \left[ P_{t+1}^{b-1} M_{t+1}^{1-b} \right] \right\}$$

or in a shorter form by:

$$Z_{t+1} = \sum_{i=1}^4 E [h_i(P_{t+1}, M_{t+1})],$$

where the function  $h_i$  takes the form:

$$h_i(P_{t+1}, M_{t+1}) = P_{t+1}^{a(i)} M_{t+1}^{b(i)}, \quad i = \overline{1, 4}$$

where  $h : R^n \times R^n \rightarrow R^n$ , and both  $a(i)$  and  $b(i)$  have the domain given by the set  $\{1, 2, 3, 4\}$  taking values in a set of the parameters of the model and algebraic combination of them denoted by  $S$ .

Thus,  $a : \{1, 2, 3, 4\} \rightarrow S$ ,  $b : \{1, 2, 3, 4\} \rightarrow S^3$ ,

$$\text{where } a(i) = \begin{cases} b-1, & i=1, 4 \\ \alpha-b, & i=2 \\ b-\alpha-1, & i=3 \end{cases}, \quad \text{and } b(i) = \begin{cases} 2b-1, & i=1 \\ 2b-\alpha-1, & i=2 \\ 1-b+\alpha, & i=3 \\ 1-b, & i=4 \end{cases}.$$

Further, the expectation variable  $Z_{t+1}$  :

$$Z_{t+1} = \sum_{i=1}^4 E [h_i(P_{t+1}, M_{t+1})] = \sum_{i=1}^4 E \left[ P_{t+1}^{a(i)} M_{t+1}^{b(i)} \right], i = \overline{1, 4}$$

which leads to:

$$Z_{t+1} = \sum_{i=1}^4 \left( E \left[ P_{t+1}^{a(i)} \right] + E \left[ M_{t+1}^{b(i)} \right] + \text{cov} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) \right), i = \overline{1, 4}.$$

The covariance term is:

$$\text{cov} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) = E \left[ z \left( \left( P_{t+1}^{a(i)} - E \left[ P_{t+1}^{a(i)} \right] \right) \left( M_{t+1}^{b(i)} - E \left[ M_{t+1}^{b(i)} \right] \right) \right) \right].$$

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<sup>3</sup>Basically,  $S = \{b-1, \alpha-b, b-\alpha-1, 2b-1, 2b-\alpha-1, 1-b+\alpha, 1-b\}$ .



The approximation of expectations operator (5.7) is applied again:

$$\begin{aligned} \text{cov} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) &= w_0 \left( P_t^{a(i)} - E \left[ P_{t+1}^{a(i)} \right] \right) \left( M_t^{b(i)} - E \left[ M_{t+1}^{b(i)} \right] \right) + \\ &+ w_1 \left[ \left( P_{t+1}^{a(i)} - E \left[ P_{t+1}^{a(i)} \right] \right) \left( M_{t+1}^{b(i)} - E \left[ M_{t+1}^{b(i)} \right] \right) \right], \end{aligned}$$

where further used in the computation of the covariance are the relations:

$$E \left[ P_{t+1}^{a(i)} \right] = w_0 z_0 \left( P_t^{a(i)} \right) + w_1 z_1 \left( P_{t+1}^{a(i)} \right)$$

and

$$E \left[ M_{t+1}^{b(i)} \right] = w_0 z_0 \left( M_t^{b(i)} \right) + w_1 z_1 \left( M_{t+1}^{b(i)} \right),$$

where  $w_0$  and  $w_1$  are as specified above and  $z : R^n \rightarrow R^n$  is the identity function.

*The expectation variable is:*

$$Z_{t+1} = \left\{ E \left[ P_{t+1}^{a(i)} \right] \right\}_{i=1}^4 + \left\{ E \left[ M_{t+1}^{b(i)} \right] \right\}_{i=1}^4 + \left\{ \text{cov} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) \right\}_{i=1}^4$$

or equivalently:

$$Z_{t+1} = \bar{E} \left[ P_{t+1}^{a(i)} \right] + \bar{E} \left[ M_{t+1}^{b(i)} \right] + \overline{\text{cov}} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right),$$

where

$$\bar{E} \left[ P_{t+1}^{a(i)} \right] = \left\{ E \left[ P_{t+1}^{a(i)} \right] \right\}_{i=1}^4, \quad \bar{E} \left[ M_{t+1}^{b(i)} \right] = \left\{ E \left[ M_{t+1}^{b(i)} \right] \right\}_{i=1}^4$$

and

$$\overline{\text{cov}} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) = \left\{ \text{cov} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right) \right\}_{i=1}^4.$$

At this stage the explicit form of the system can be written as:

$$E \left[ P_{t+1} \right] = f \left( E \left[ P_{t+1} \right], \bar{E} \left[ P_{t+1}^{a(i)} \right], P_t, E \left[ M_{t+1} \right], M_t, \bar{E} \left[ P_{t+1}^{a(i)} \right], \bar{E} \left[ M_{t+1}^{b(i)} \right], \overline{\text{cov}} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right); \theta \right)$$

or

$$E \left[ P_{t+1} \right] = f \left( \tilde{E} \left[ P_{t+1} \right], P_t, \tilde{E} \left[ M_{t+1} \right], M_t, \overline{\text{cov}} \left( P_{t+1}^{a(i)}, M_{t+1}^{b(i)} \right); \theta \right),$$

where

$$\tilde{E} \left[ P_{t+1} \right] = \left\{ E \left[ P_{t+1} \right], \bar{E} \left[ P_{t+1}^{a(i)} \right] \right\}, \quad \tilde{E} \left[ M_{t+1} \right] = \left\{ E \left[ M_{t+1} \right], \bar{E} \left[ M_{t+1}^{b(i)} \right] \right\}.$$

The model in its explicit form is:

$$E [P_{t+1}] = f(E [P_{t+1}], \bar{E} [P_{t+1}^{a(i)}], P_t, \tilde{E} [M_{t+1}], M_t, \overline{cov} (P_{t+1}^{a(i)}, M_{t+1}^{b(i)}); \theta). \quad (5.14)$$

At this stage, we are able to move on to the next step which constructs the equilibrium condition.

### 5.4.3 Step 2

In order to solve the non-linear stochastic model (5.14), the iterative numerical method of Jacobi over-relaxation presented in the previous section (5.3.2) of this chapter is applied. Starting from an initial guess  $E [P_{t+1}]^0 = P$ , each iteration  $j$ ,  $j = \overline{1, J}$  generates a new vector of values  $E [P_{t+1}]^j$  based on the previous iteration.

Convergence is achieved when

$$\max^i \left| E [P_{it+1}]^j - E [P_{it+1}]^{j-1} \right| < \varepsilon,$$

where  $i = \overline{1, 202}$  is the index of an element in the vector  $E [P_{t+1}]$  and  $\varepsilon$  is a chosen precision.

The initial guess is given by the steady state. To find the value of  $cov [M_{t+1}, P_{t+1}]$  at steady state, the model is initially solved assuming that there is no correlation between the state and response variables, the money supply and the prices,  $M_{t+1}$  and  $P_{t+1}$ , first under the assumption of perfect credibility/foresight (this is the non-linear model with rational expectations) which corresponds to  $w_1 = 0$  and then assuming absolute lack of credibility  $w_0 = 1$ .

Given the outcomes of these experiments, assuming that the economy behaves in the two extreme cases of perfect and then absolute lack of credibility, the starting value covariance is constructed according to (5.8), where the expected value of the price  $E [P_{t+1}^{a(i)}]$  is calculated according to (5.7) given that at the first iteration,  $z_0(P_t^{a(i)}) = P_t$  obtained under the first assumption and  $z_1(P_t^{a(i)}) = P_t$  under the second one.

After the settings for the initial point given under steady state are realized, the iterative process starts and continues until a fixed point is reached:

$$E [P_{t+1}^{\max}]^j = \phi f \left( \tilde{E} [P_{t+1}^{\max}]^{j-1}, P_t, \tilde{E} [M_{t+1}], M_t, \overline{cov} (P_{t+1}^{a(i)}, M_{t+1}^{b(i)})^{j-1}; \theta \right)$$

$$+ (1 - \phi) E [P_{t+1}^{\max}]^{j-1}$$

During this process, at each new iteration, the price that maximizes the profit is found and the new covariances, based on the outcome of the last iteration are calculated. The process stops when the difference between two consecutive maximizing profit prices is less than a given precision. The last price calculated is the one retained.

Given this, the output vector  $Y_t$  can now be calculated using again:

$$Y_t = \frac{M_t}{P_t},$$

where  $M_t$  is the money supply announced by the monetary authority and  $P_t$  is the one that has just been determined.

It is important to stress that in the case of perfect foresight the expectation variables are redundant as are the covariance terms, so the solution to the model under perfect foresight requires only the second of these steps<sup>4</sup>.

## 5.5 Conclusion

This chapter explains the framework and the methodology employed to solve the model described in Chapter 3 in which, the case of imperfect credibility described in Chapter 4 was incorporated. So far, in the literature work has been done only considering the case of perfect foresight.

After presenting a general modelling framework described by an equilibrium condition, an expectation variable and an expectation updating rule, a more specific framework, representative for the model discussed in this thesis becomes the focus of attention. This allows to explain the way in which the model has been solved. This was achieved in two steps. The first step entails mapping the non-linear stochastic problem into a form more convenient to work with numerically. The second step constructs the equilibrium solution to that simpler problem. Given that for the case of perfect foresight the expectation variables are redundant as are the covariance terms, the solution to the model under perfect foresight requires only the second of these steps.

Finally, the solution method described was applied to solve the model of this thesis

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<sup>4</sup>A computational solution procedure in *MatLab* has been developed to solve the model applying this solution.

(described in Chapter 3 and adjusted with the imperfect credibility of Chapter 4) and we discussed in detail the solution procedure for the case of single price strategy. A third step has been added in order to allow the model to be written in a structural form and specifically, in terms of the state, response and expectation variables.

After presenting the model chosen to address the issue of imperfect credibility in a transition to price stability in Chapter 3, the way imperfect credibility has been modelled in Chapter 4, and the methodology employed to solve this new model in this chapter, attention can be finally focused on the analysis of the behaviour of both output and the optimal speed of disinflation during a disinflationary process when there is a temporary lack of credibility. These will be discussed in depth in the next two chapters.

# Chapter 6

## Output Effects of Disinflation under Perfect and Imperfect Credibility

### 6.1 Introduction

At this stage, the main aim of this thesis, namely the analysis of the behaviour of output during a disinflation policy and that of the optimal speed of disinflation in conditions of imperfect credibility, in an economy like that described in Chapter 3, can be finally pursued. The focus of this chapter is on the behaviour of output.

As presented in Chapter 2, recent research by Ireland (1997), Ball (1994), King and Wolman (1999), Khan, King and Wolman (2003) has discussed the effects of monetary contractions in an economy in which the supply side of the economy is characterised by monopolistically competitive firms and in which there is rigidity in the settings of prices, under the assumption of perfect foresight. The main results arising from this work significant to this thesis are:

1. In the periods following a contraction in the money stock, real output is likely to fall below its long-run equilibrium level.
2. A gradual disinflation may actually result in output, after its initial decline, rising above its new steady-state level, and remaining high for some time.

Although literature discussing the behaviour of output under both cases of perfect and imperfect credibility is available, no research has been conducted in a setup for imperfect credibility like the one described in Chapter 4. This setup allows the examination of the behaviour of output during a disinflation policy under a temporary lack

of credibility in a more complex, but now standard economy as previously described in Chapter 3.

Therefore, the two above presented results from the existing literature can now be explored in such conditions and the behaviour of output after a monetary contraction when the price setters are assumed to learn in various ways about the government's announcement of the money supply can be analysed. This setup led to some computational complexities related to the optimal choice of prices by firms who not only have to forecast future demand and cost conditions, but also have to forecast their covariances, as seen in the previous chapter.

This chapter will present an analysis of the behaviour of output during a disinflationary process, under both perfect and imperfect credibility, focusing on the latter. First an analysis of output's behaviour during a disinflationary process in an economy under perfect foresight as in King and Wolman (1999) and Ireland (1997) will be presented, followed by a presentation of the case in which the disinflation process takes place in an environment with (temporarily) imperfect credibility. Under imperfect credibility, under each of the expectations updating discussed, the learning process is determined, as presented in Chapter 4, by: the initial degree of credibility of the private sector in the government announcement, the length of the learning period over which perfect foresight is achieved and the type of learning process itself (concave or convex). Therefore, an attempt will be made to identify the output effects of each of these factors and to quantify them when they appear. This is achieved by comparing various (simulated) paths of output during disinflation which are obtained by keeping all the other above mentioned factors that determine the learning process constant, and varying the factor under study.

While trying to study the effects of various factors that influence output's behaviour, a distinction will be made between the elements that define the settings for the study of a disinflationary process such as: the length of the period of disinflation and the initial annual inflation rate, and those that establish the settings for a learning process such as: the length of the learning period over which perfect foresight is achieved, the initial level of lack of credibility and the type of learning. All these were presented in detail in Chapter 4. Moreover, each of these factors will be studied under both the expectation updating rules (4.3) and (4.4) presented in section 4.2.1.

The outline of the chapter is as follows: the behaviour of output under the case of perfect foresight is presented in Section 6.2. Section 6.3 presents the behaviour of output under (temporary) imperfect credibility. This section presents the output effects of each factor involved in defining of the learning process. Subsection 6.3.1 presents the output effect of the type of learning, followed in 6.3.2, 6.3.3, 6.3.4 by the output effects of the expectation updating, the length of the learning period over which perfect foresight is achieved and the initial level of lack of credibility respectively. Section 6.4 concludes.

## 6.2 Output Effects of Disinflation under Perfect Credibility

The results presented in this section<sup>1</sup> are taken from Ireland (1997) and are realized in an economy under perfect foresight. He presents the behaviour of output under the cases of an immediate and a gradual disinflation (over 3 years) of a small (3%) and a high (200%) initial annual inflation rate. Looking at the evolution of the output under all these cases in order to minimize the output loss, he concludes that under perfect foresight small inflations are best disinfated gradually, while big inflations are best disinfated immediately. Disinflatng a small inflation immediately brings a big output loss. However, the output loss is minimized if the disinflation is more gradual. For the case of a high inflation, the results are the opposite.

Figure 6-1 shows the effect of an immediate disinflation ( $T = 1$ ) (see (3.19)) on output when the initial annual inflation rate is 3% (the dotted line). From here it can be observed that at relatively low rates of inflation, immediate disinflation is quite costly. The initial fall in output is almost 1.47% and this persists for two consecutive periods of time, as firms follow a single price strategy. The 'hump-shaped' response is due to the fact that the first set of firms, in order to set new prices, increases their prices. Past inflation has eroded their relative real price and now they face a relatively large increase in demand for their products (since the firms that do not re-price have relatively high prices and hence relatively low demand).

The next case presented is the gradual disinflation of the same initial annual infla-

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<sup>1</sup>replicated by the author of the thesis for the purpose of this research.

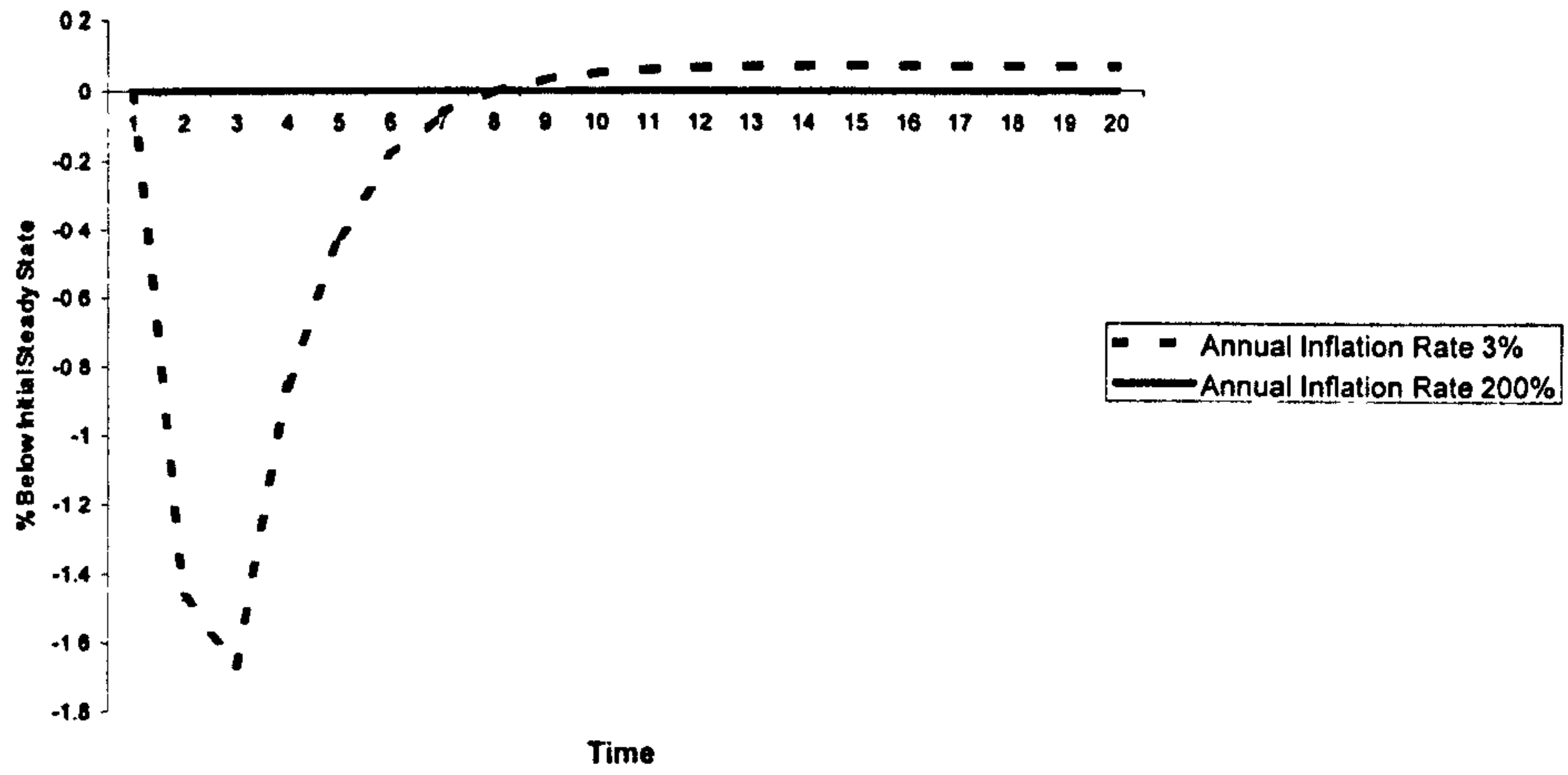


Figure 6-1: Output Effects of Immediate Disinflation under Perfect Credibility. Initial Annual Inflation Rates 3% and 200%

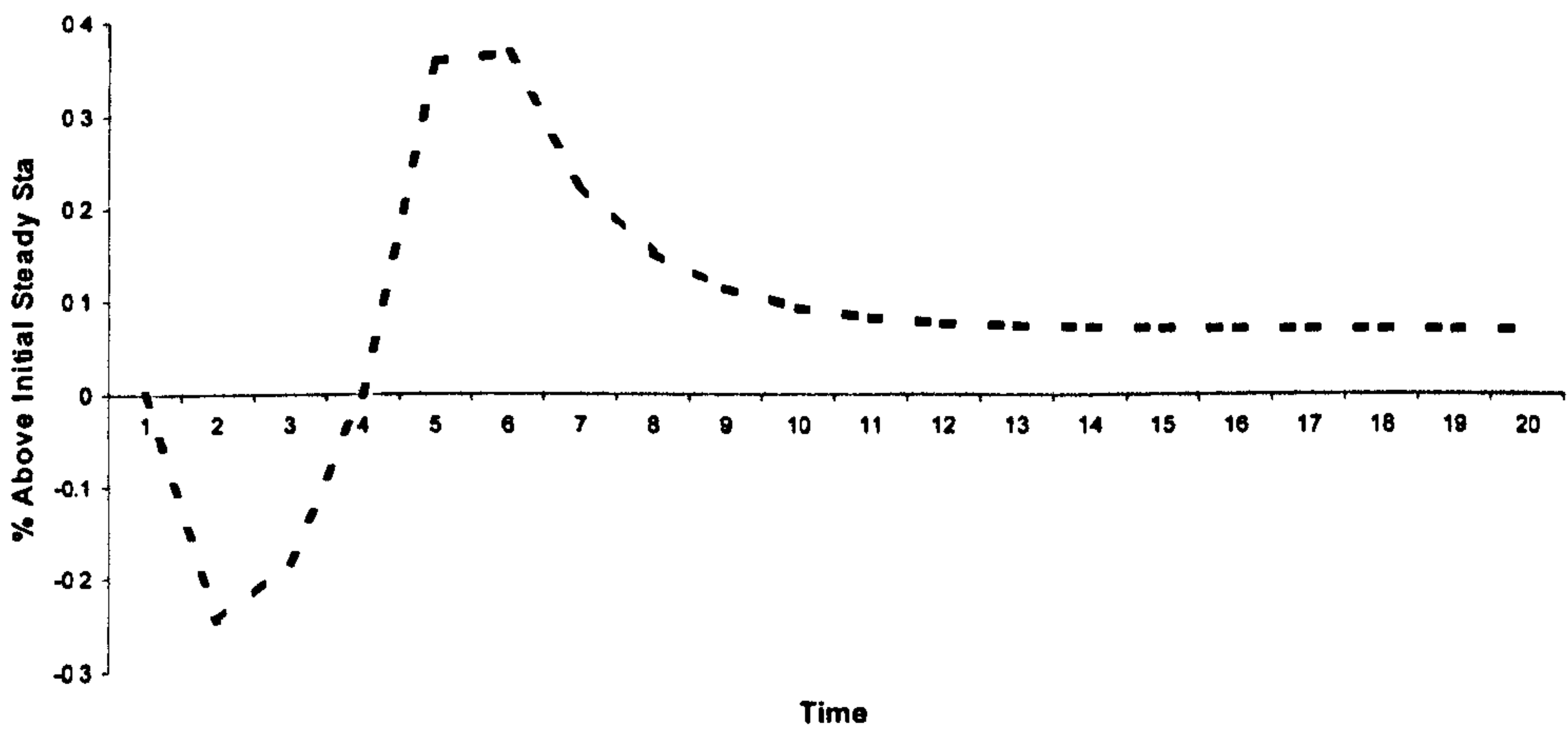


Figure 6-2: Output Effects of Gradual Disinflation under Perfect Credibility. Initial Annual Inflation Rate 3%. T=6



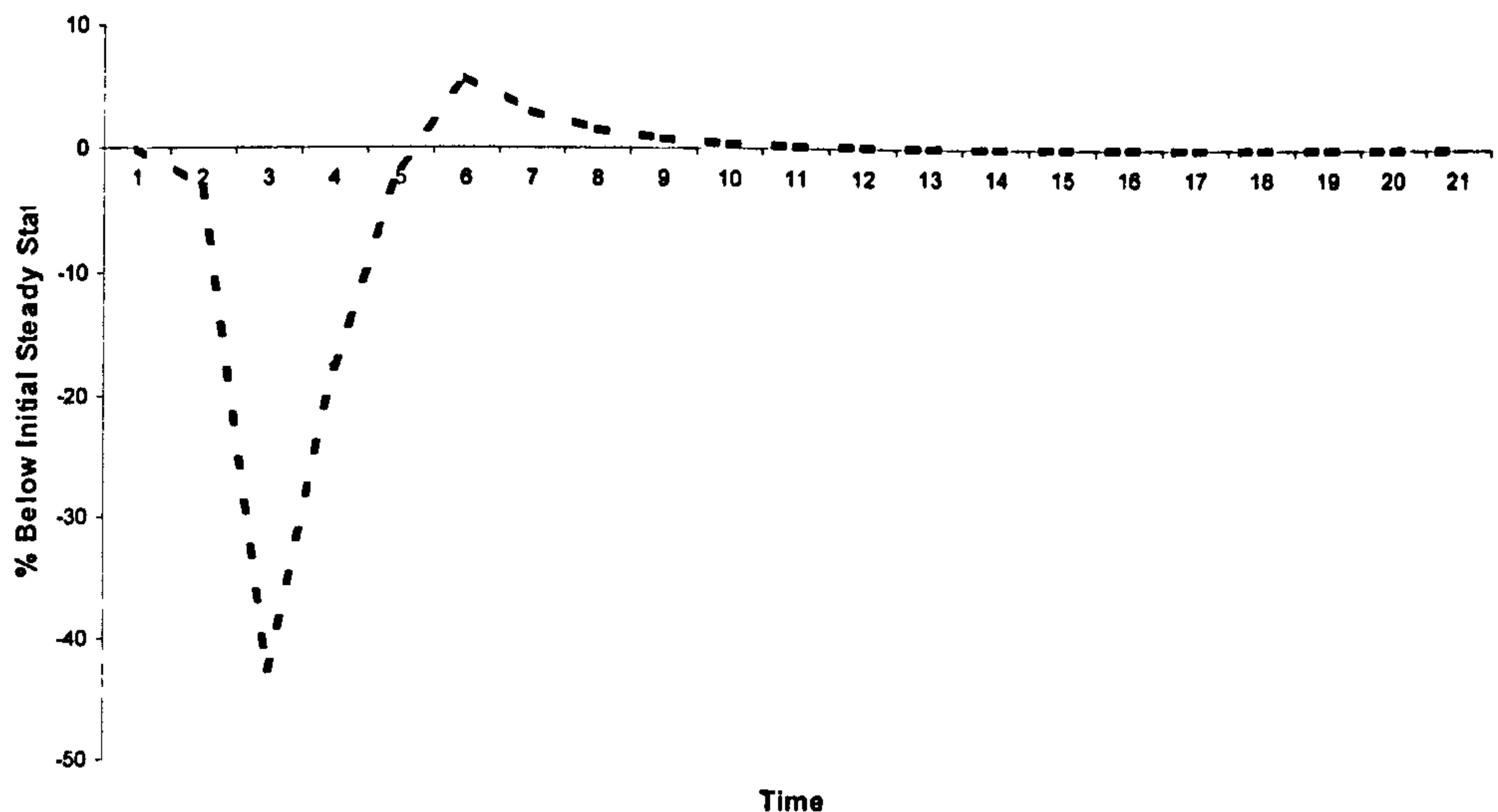


Figure 6-3: Output Effects of Gradual Disinflation under Perfect Credibility. Initial Annual Inflation Rate 200%.  $T=6$ .

tion rate of 3%, when the disinflation takes place over a period of 3 years ( $T = 6$ ). The path of output is plotted in figure 6-2. Here there are two striking things:

1. The fall in output is much smaller. In the first period, after the disinflation process is announced, the fall in output is only 0.24%, compared to the perfect foresight case where the output falls to 1.47% below the initial steady state. The output loss, (measured by the area between the curve describing the output behaviour and the initial output steady state, the  $0x$  axis) is obviously smaller than under the immediate disinflation case.
2. Moreover, we observe that after period 3, which corresponds to one and a half years, a disinflationary boom appears<sup>2</sup>. The explanation given for these booms so far is that under the case of perfect credibility, agents are behaving in advance of the change in policy, lowering their relative prices, knowing that in future, the inflation is going to be lower. Agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, causing a boom.

Furthermore, after an initial drop in output, a gradual disinflation leads to a disinflationary boom for a relatively prolonged period but also to a new steady state for

<sup>2</sup>The disinflationary booms were first observed by Ball (1994) and King and Wollman (1999).

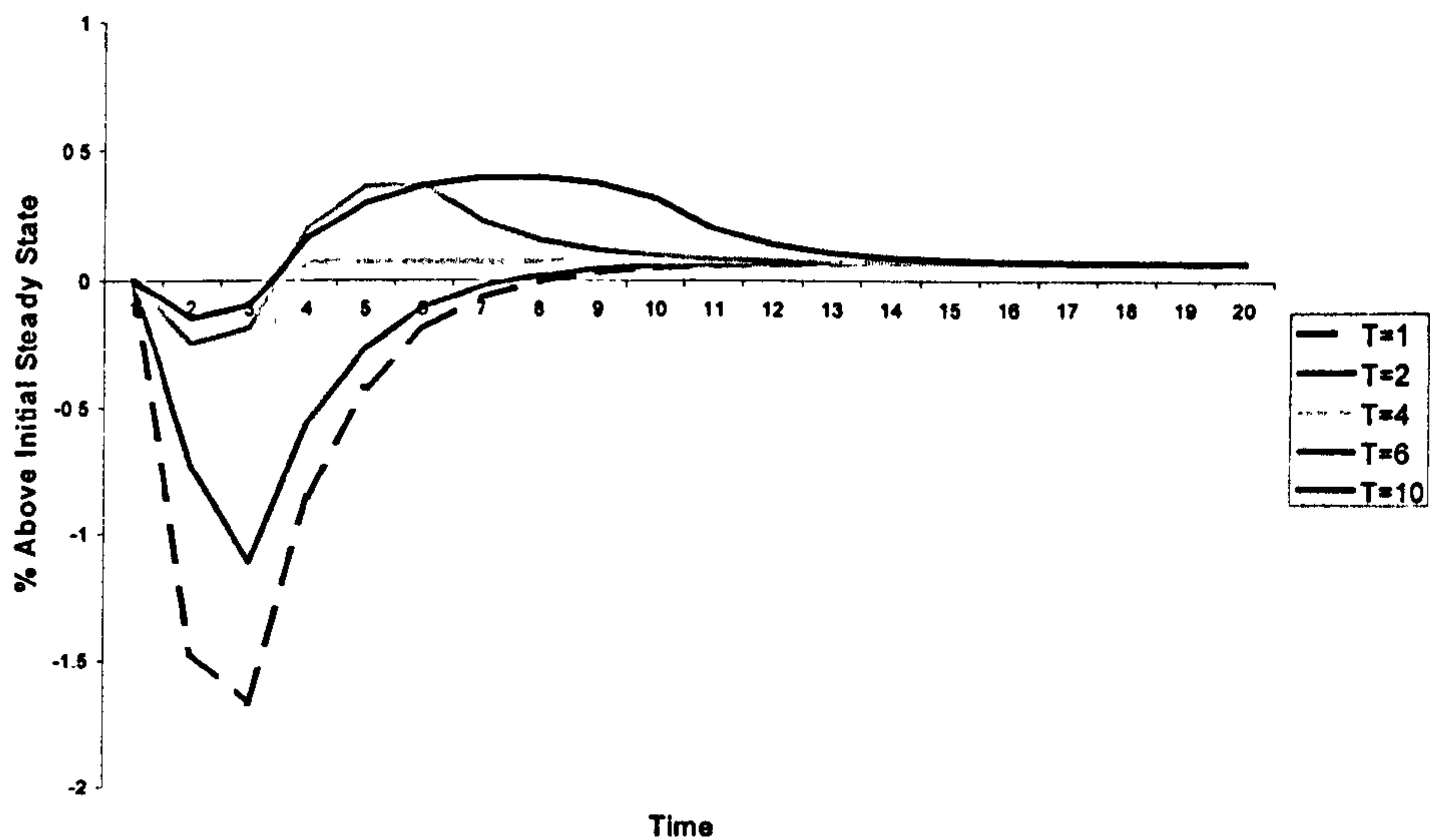


Figure 6-4: Output Effects of Gradual Disinflation under Perfect Credibility. Initial Annual Inflation Rate 3%.  $T$  from 1 to 10.

the output which is above the initial one. So far, by comparing figure 6-1 to figure 6-2 it can be seen that small inflations are best disinfated gradually.

Looking at more gradual disinflations, that takes place over 1, 2, 3 and 5 years ( $T = 2$ ,  $T = 4$ ,  $T = 6$ ,  $T = 10$ ), it can be seen in figure 6-4 that a more gradual approach to disinflation results in a correspondingly small output loss.

The immediate disinflation of a 200% initial annual inflation rate, plotted in figure 6-1 (the full line) shows that at higher rates of inflation, disinflation can proceed with no relative-price distortion. This is due to the fact that in an environment with high inflation, firms re-price every period adopting the two price strategy described in 3.3.2. However, this is not the case at all for a gradual disinflation which takes places over 3 years. As seen in figure 6-3, here there is considerable loss in output.

This chart shows that at high initial rates of inflation, the loss of output in the initial periods can be substantial if a gradual disinflation is adopted (in this case of 3 years). This is explained by the fact that at high rates of inflation firms do not initially change their prices and end up with big output losses.

To conclude, three main issues must be kept in mind:

1. In order to be realized with minimal output losses, small inflations are best disinfated gradually, while big inflations are best disinfated immediately.

2. In the periods following a contraction in the money stock, real output is likely to fall below its long-run equilibrium level.
3. A gradual disinflation may actually result in output, after its initial decline, rising above its new steady-state level, and remaining high for some time.

In data, periods of disinflation are rarely accompanied by such booms. Therefore, naturally, these results raise the question of how output will be affected by (temporary) imperfect credibility during a disinflation process. More specifically, the question raised is of whether the output boom will still appear or not. If the answer is positive, then the focus moves onto what determines its appearance. Consequently, the output effect of each of the factors determining the learning process will be analysed.

### 6.3 Output Effects of Disinflation under Imperfect Credibility

This section presents an analysis of the behaviour of output under a disinflationary process when there is not perfect credibility but when the agents are learning over time and eventually achieve perfect foresight. The output effects of each of the factors determining the learning process: the length of the learning period over which perfect foresight is achieved, the initial level of lack of credibility and the type of learning are discussed. Needless to say, only the two non-linear types of learning will be considered.

In comparing *the two types of non-linear learning* under each of the expectation updating (4.3) and (4.4), the conclusion is that the way the private sector learns at the beginning of the change in policy is pivotal: disinflation under concave learning is more costly, as agents initially adjust slowly to the new announced path of money supply. Under convex learning agents avoid some of the more costly mistakes early on in the disinflation compared to concave learning.

Furthermore, depending on *the expectation updating*, the output effects of the type of learning are minimized, especially under the second expectation updating. The cost of disinflation under concave learning is reduced under the second rule, since the initial expectational errors are much reduced. For this reason, under the second rule, the concave paths of output are extremely close to those seen in the case of perfect

foresight. This result suggests that the disinflationary booms are more likely to occur under (4.4) while, convex learning tends to make these booms disappear.

Regarding *the length of the period of time over which learning is achieved*, the longer this is, the longer and more severe the recession will be. This is valid under either concave or convex type of learning, regardless of the expectation updating. Evidently, the recession/output loss is much bigger under the concave learning. Also, it can be observed that for learning periods greater than 5 years and for small inflation rates, output displays a cyclical behaviour. Although the (negative) output effects of the fact that the public is reluctant to believe the government's announcement at the beginning of the regime change appear to be minimized under the second expectation updating, these are propagated into the future appearing later also due to an overlapping effect. Another aspect is that a high level of credibility *at the beginning* of the change in policy minimizes the output losses.

The higher the level of credibility of the private sector at the beginning of the change in policy, the smaller the cost of the disinflation is in terms of output loss. Second, the effects of a learning type on the cost of disinflation dominate those due to a low level of credibility. This means that even if there is absolute lack of credibility at the moment of the regime shift, if the policymakers can gain the private sector's trust by making the policy more credible to the private sector (by for example increasing transparency and/or communication) disinflation can be realized with smaller output loss. If the manner in which the expectations are formed can be influenced to make the private sector learn faster (to achieve faster perfect foresight), then this can make a change to the cost of disinflation. Influencing the manner in which expectations are formed in the right direction can also lead to a shorter period of time over which the private sector is learning. It has also been established in this research that the shorter the learning process, the better.

### **6.3.1 The Output Effect of the Type of Learning**

This subsection investigates the effect of the type of learning during a disinflation process on the path of output. The output effects of the type of learning are studied under the case of a gradual disinflation realized over 3 years ( $T = 6$ ) of the same initial annual inflation rate of 3%, considering that perfect foresight is achieved in 3 years

( $N = 6$ ). It is also assumed that at the beginning of the regime change, the learning process starts from an absolute lack of credibility of the private sector ( $\rho_0 = 1$ ). The investigation is first undertaken for the expectation updating rule (4.3) and secondly for (4.4). These will be referred to in the figures present in this thesis as R1 and R2 respectively. The results are compared in the first instance to those obtained under the case of perfect foresight and then are compared to each other.

In order to provide a complete discussion of the output effect of the type of learning, first these are analysed separately under each of the two expectation updating rules presented in this thesis, in section 6.3.1.1 (first rule (4.3)) and in section 6.3.1.2 (second rule (4.4)).

#### 6.3.1.1. Expectation Updating (4.3)

As presented in section 4.1.1, the first expectation updating rule considered is when agents perceive the authorities to be reverting to the previous steady state inflation. The concave learning presented in section 4.2.3, captures the case in which agents are reluctant at the beginning of the change in policy to believe the announcement of the monetary authority, but as time goes by they are more willing to accept it. This subsection investigates the implications of this type of learning on the behaviour of output when disinflation is implemented immediately and then gradually.

For the case of an *immediate disinflation* of a 3% initial annual inflation rate, figure 6-5 shows that the output in the second period of time falls even more under concave learning in comparison with the case of perfect foresight. Also, due to the fact that the learning process takes time, the output loss is bigger. As discussed in section 6.2, as a small inflation is best disinflated gradually, an immediate disinflation under the case of imperfect credibility is not desirable.

For the case of a *gradual disinflation*, figure 6-6 compares the path of output under concave learning to that obtained under the case of perfect foresight for the same initial inflation rate of 3%. The contraction in output is more pronounced and more protracted under imperfect credibility. The output keeps falling in the second period as well. And, even though by period 6 (3 years) agents in both economies have the same information, the effect of imperfect credibility remains for some time, due to the overlapping nature of price setting, under the initial steady state level.

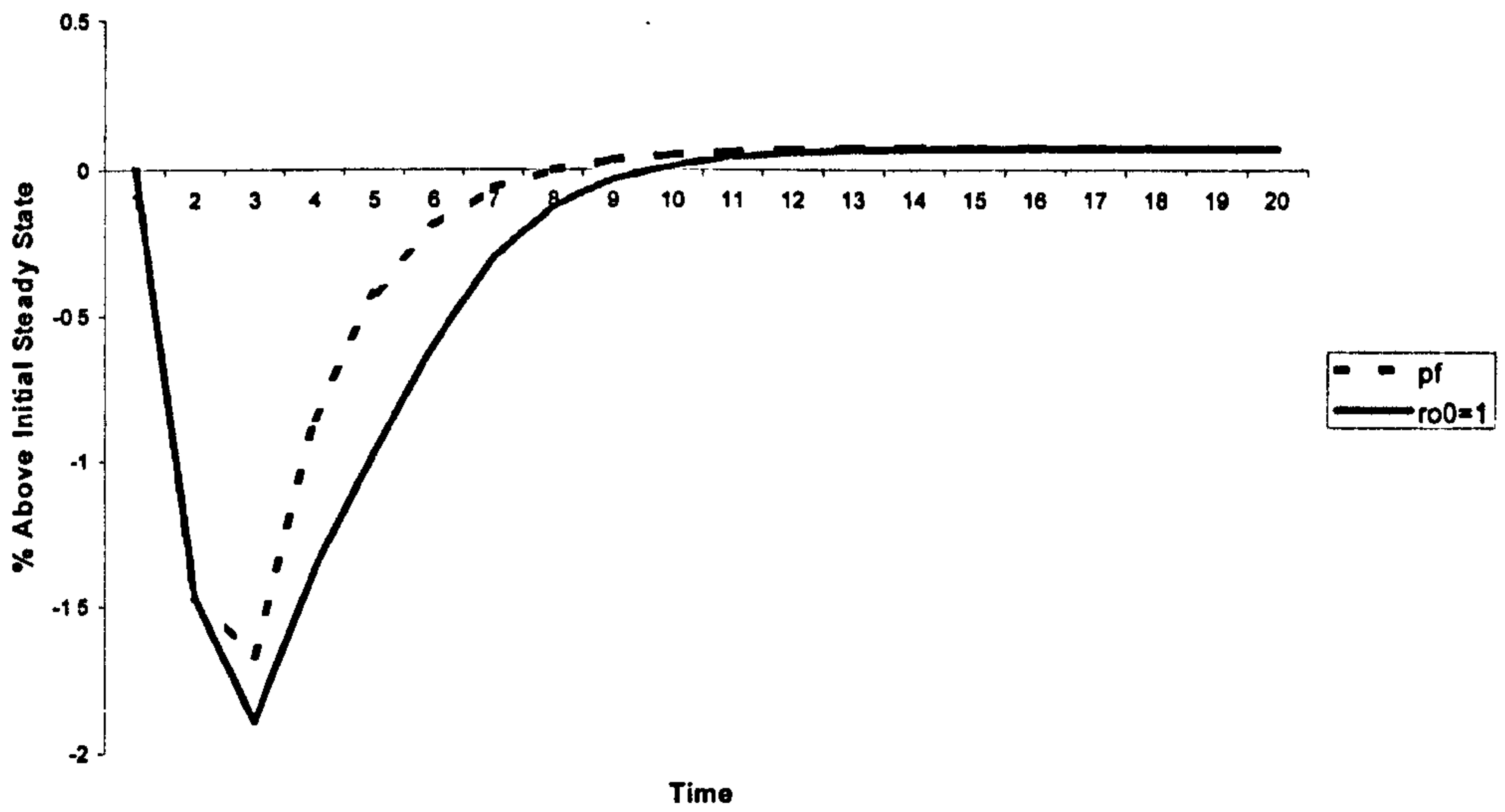


Figure 6-5: Output Effects of Immediate Disinflation under Concave Learning. Initial Annual Inflation Rate 3%. 3 Years to Perfect Foresight. R1

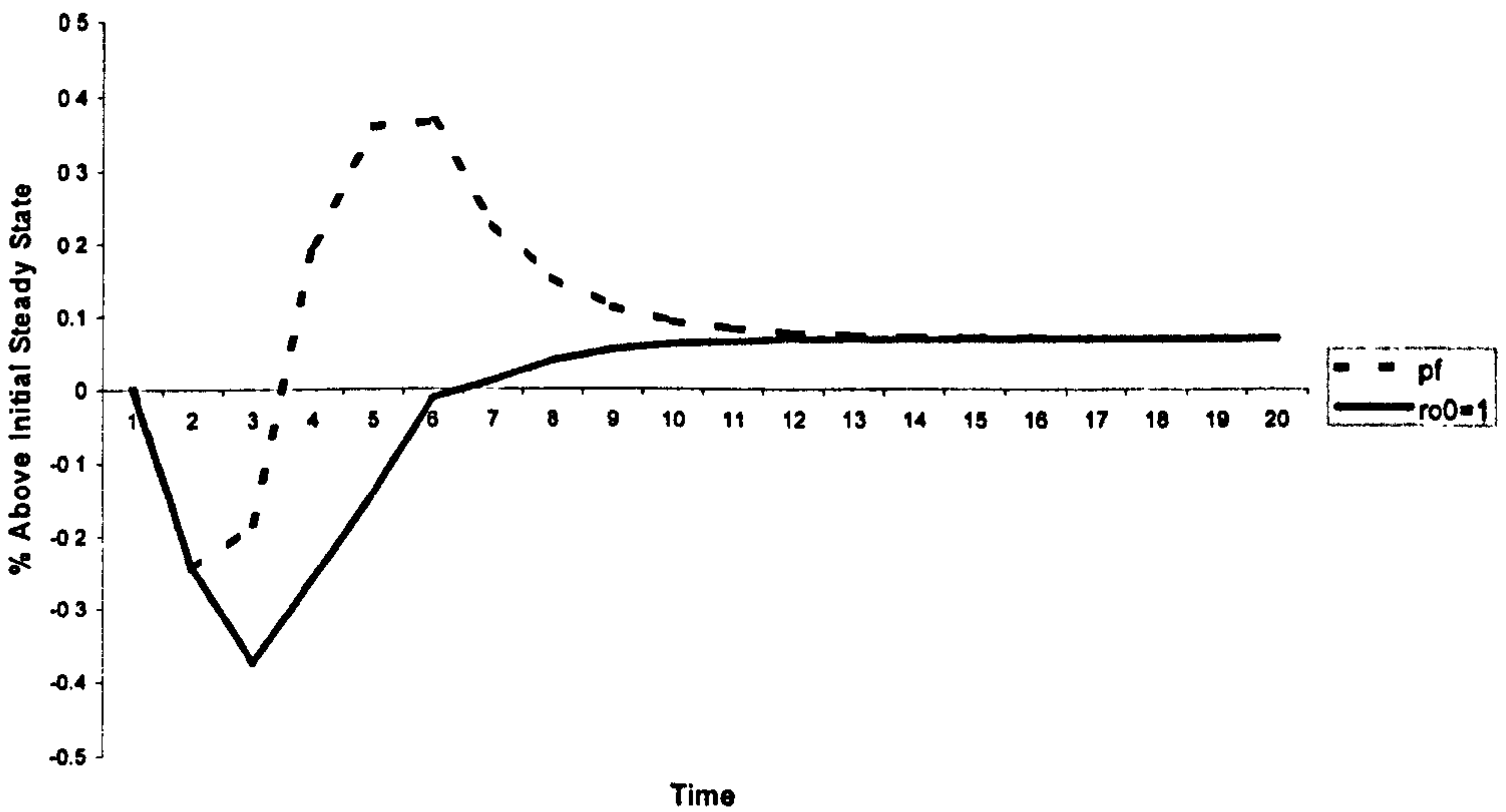


Figure 6-6: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%. T=6. 3 Years to Perfect Foresight. R1

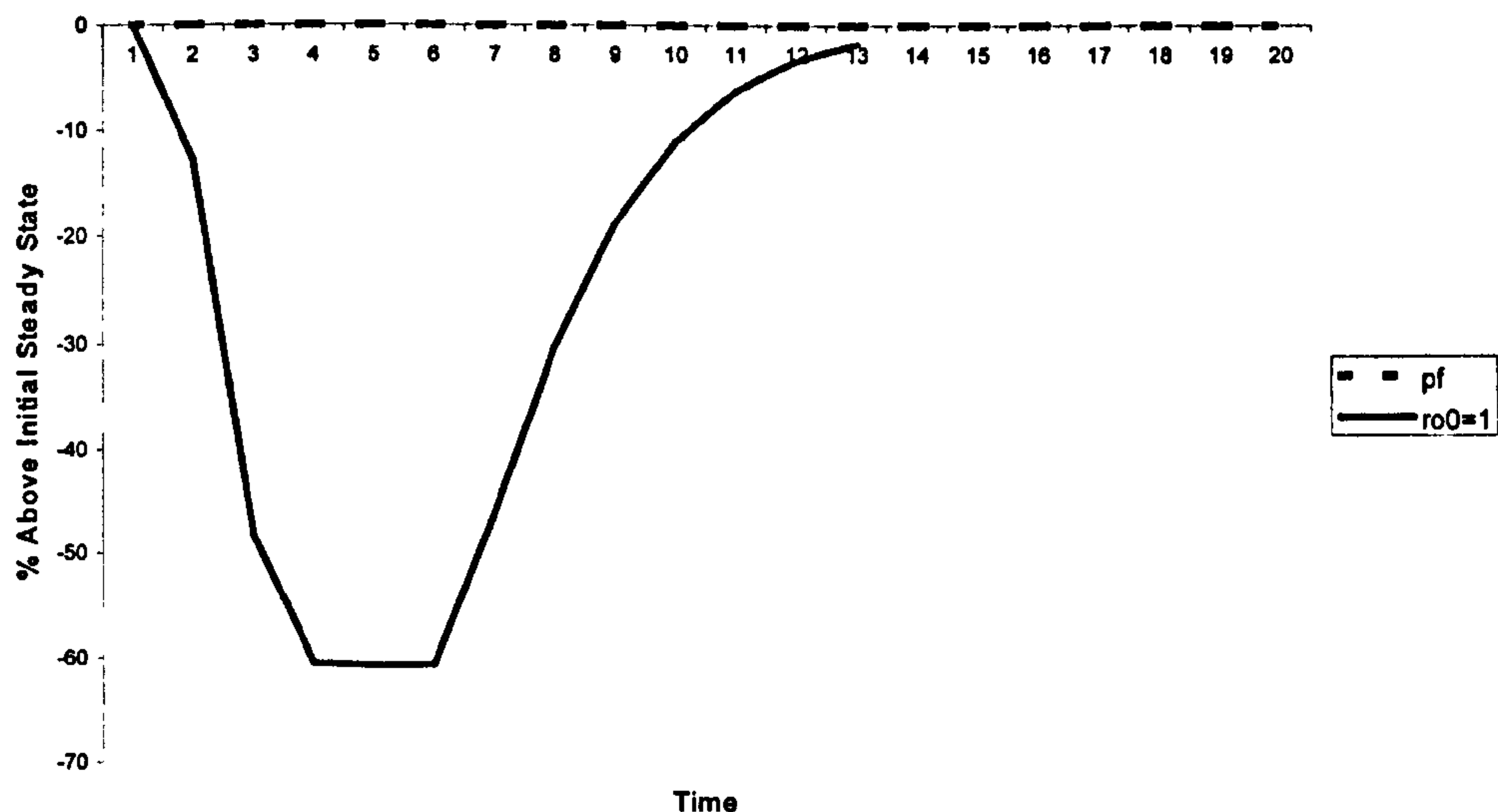


Figure 6-7: Output Effects of Immediate Disinflation under Concave Learning. Initial Annual Inflation Rate 200%. 3 Years to Perfect Foresight. R1

One of the potential counterfactual implications of the case of perfect foresight is the 'disinflationary boom' or the tendency for output to rise above its new steady state level under a gradual disinflation as agents anticipate lower future price-levels. Figure 6-6 shows that under concave learning this effect vanishes as output falls more sharply and does not rise above its new steady state value along the transition path. Agents only gradually come to realize that the price-level is growing at a zero rate, a realization that is delayed due to the gradual nature of the disinflationary process itself.

For very high initial inflation rates, the fall in output following an immediate disinflation is catastrophic, as figure 6-7 demonstrates. It is also of a similar order of magnitude under a more gradual disinflation, as plotted in figure 6-8. Here the output effect of the concave learning is much enhanced when there is a high inflation: the fact that the private sector is reluctant to believe the government's announcement at the beginning of the change in policy, has a considerable effect on output. However, this is diminished under convex learning, but still far from acceptable.

The convex learning, presented in section 4.2.4, reflects the learning path of a population who, although happy to accept that the monetary authority also dislikes the current, relatively high, rate of inflation, they worry that as the slope of the short-run Phillips curve flattens, the monetary authority may be tempted to renege. Figure 6-10

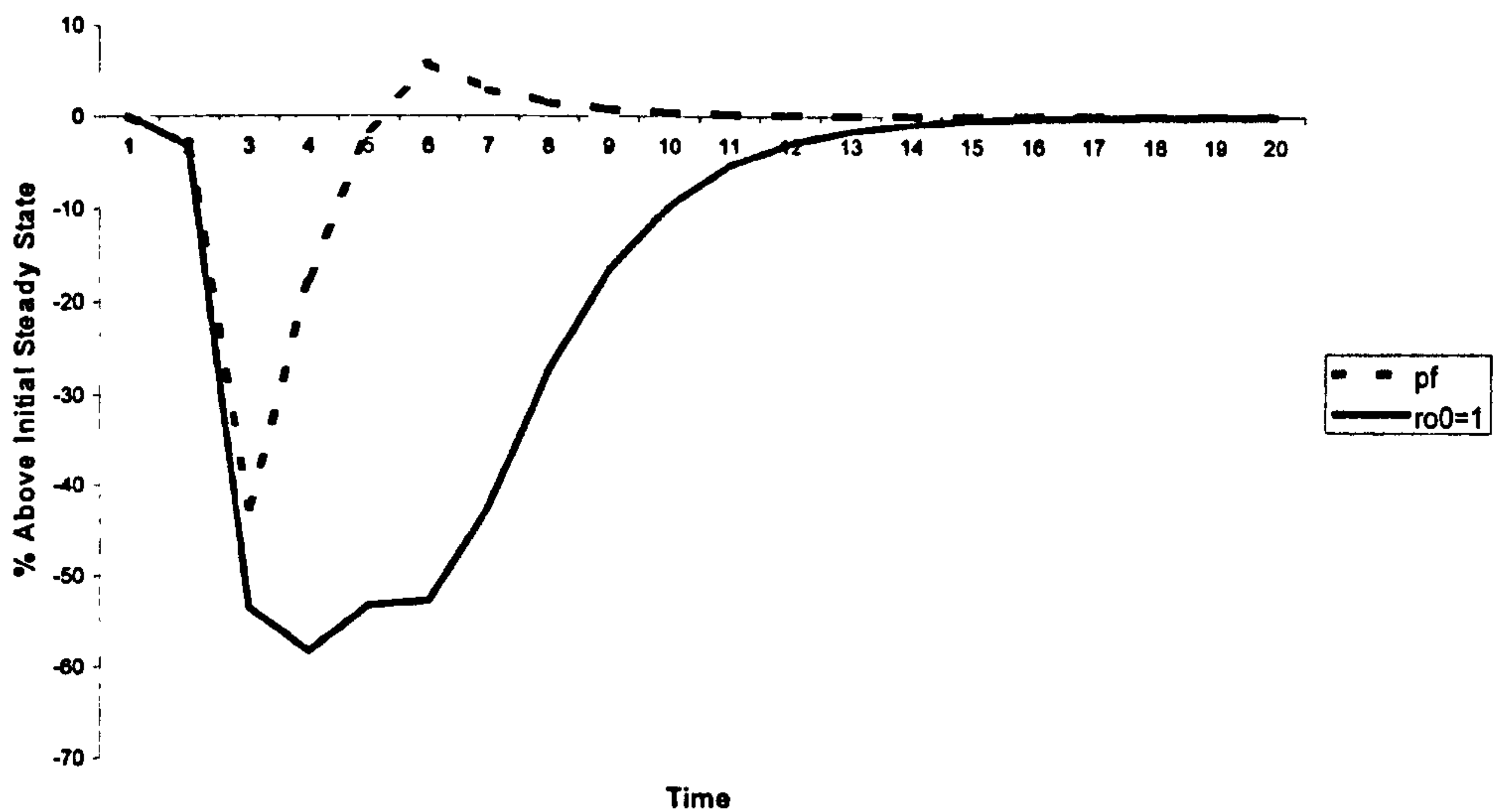


Figure 6-8: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 200%.  $T=6$ . 3 Years to Perfect Foresight. R1

shows that the drop in output under immediate disinflation under convex learning leads to a drop in output more severe than, but close to, the drop under perfect foresight.

Agents tendency to believe the authorities when they announce a decrease in the money growth rate but also to become more reluctant to do so as time goes by, allows the disinflationary booms to occur. It can be observed that for 'moderate' rates of inflation such booms are absent, whether the disinflation is immediate (figure 6-10) or gradual (figure 6-11).

Comparing the output effects of concave and convex learning under the first expectation updating rule it is apparent from the figures presented so far that, regardless of the initial annual inflation rate, the output under convex learning closely resembles that of perfect foresight. The reason for this is that the convex path of  $\rho_t$  means that agents avoid some of the more costly mistakes early on in the disinflation that occur under concave updating. This means that the credibility of the private sector at the beginning of the announcement of the change in policy plays a major role.

A boom in output still occurs even under the case of imperfect credibility, when the private sector is willing to believe the announcement of the monetary authority at the beginning of the regime change. This holds even though with time, agents become



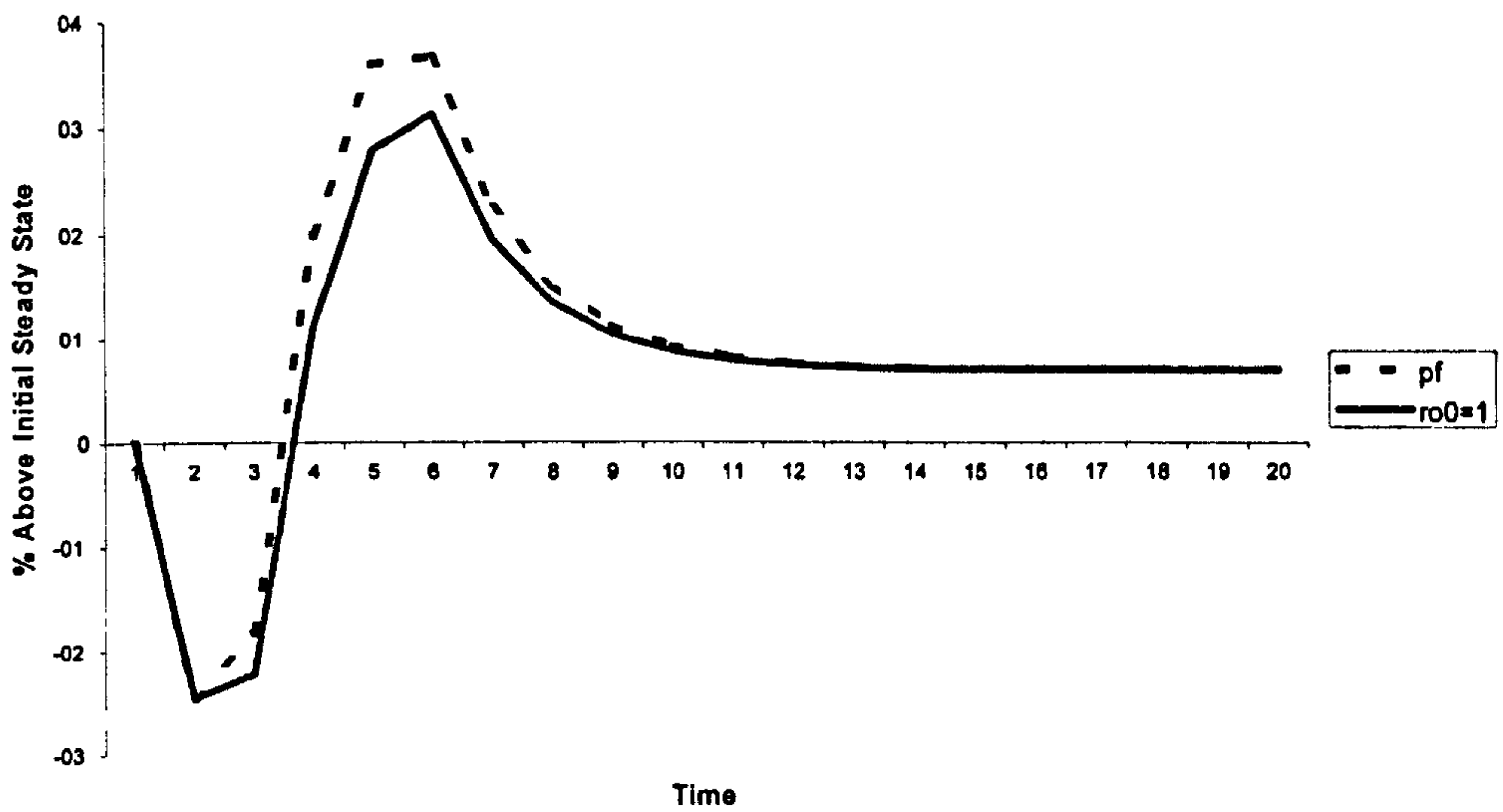


Figure 6-9: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1

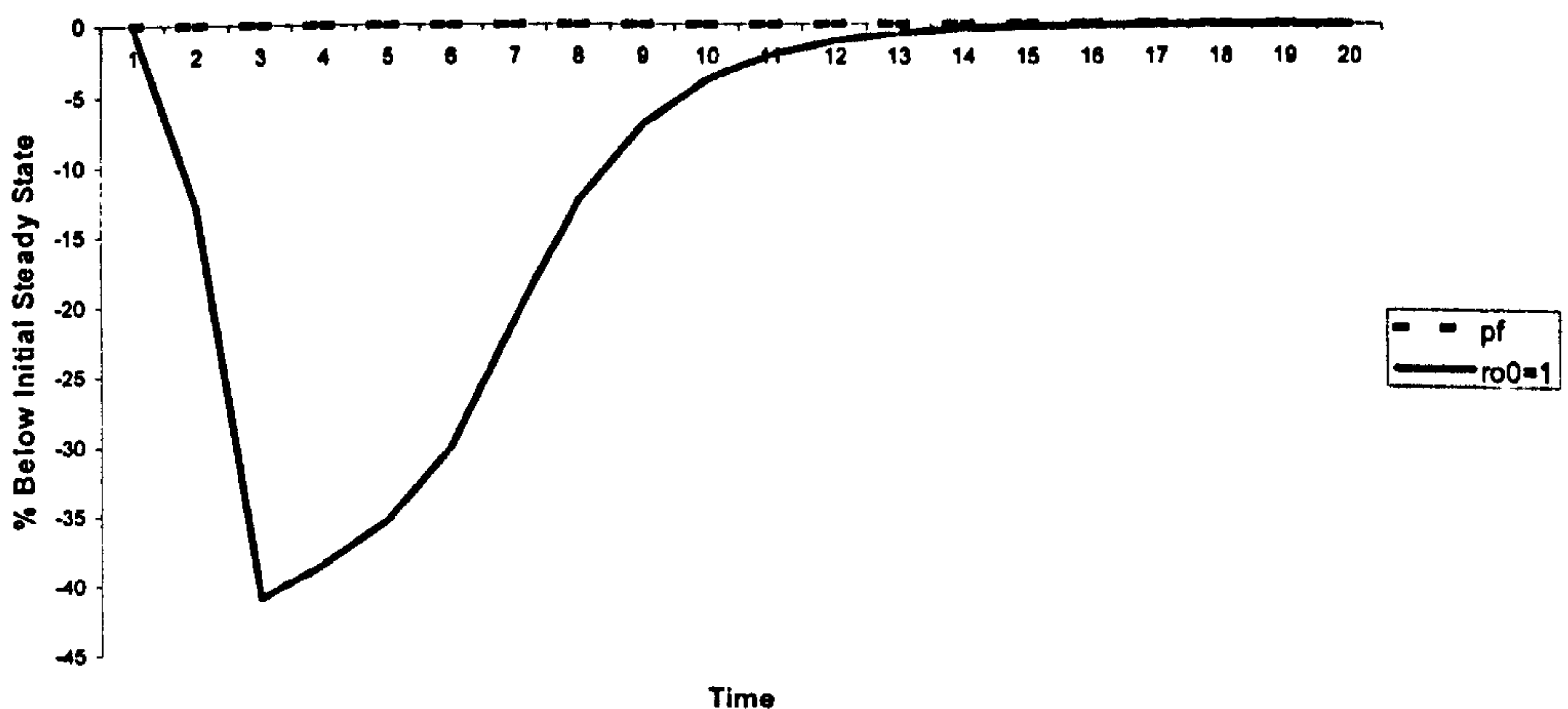


Figure 6-10: Output Effects of Immediate Disinflation under Convex Learning. Initial Annual Inflation Rate 200%. 3 Years to Perfect Foresight. R1

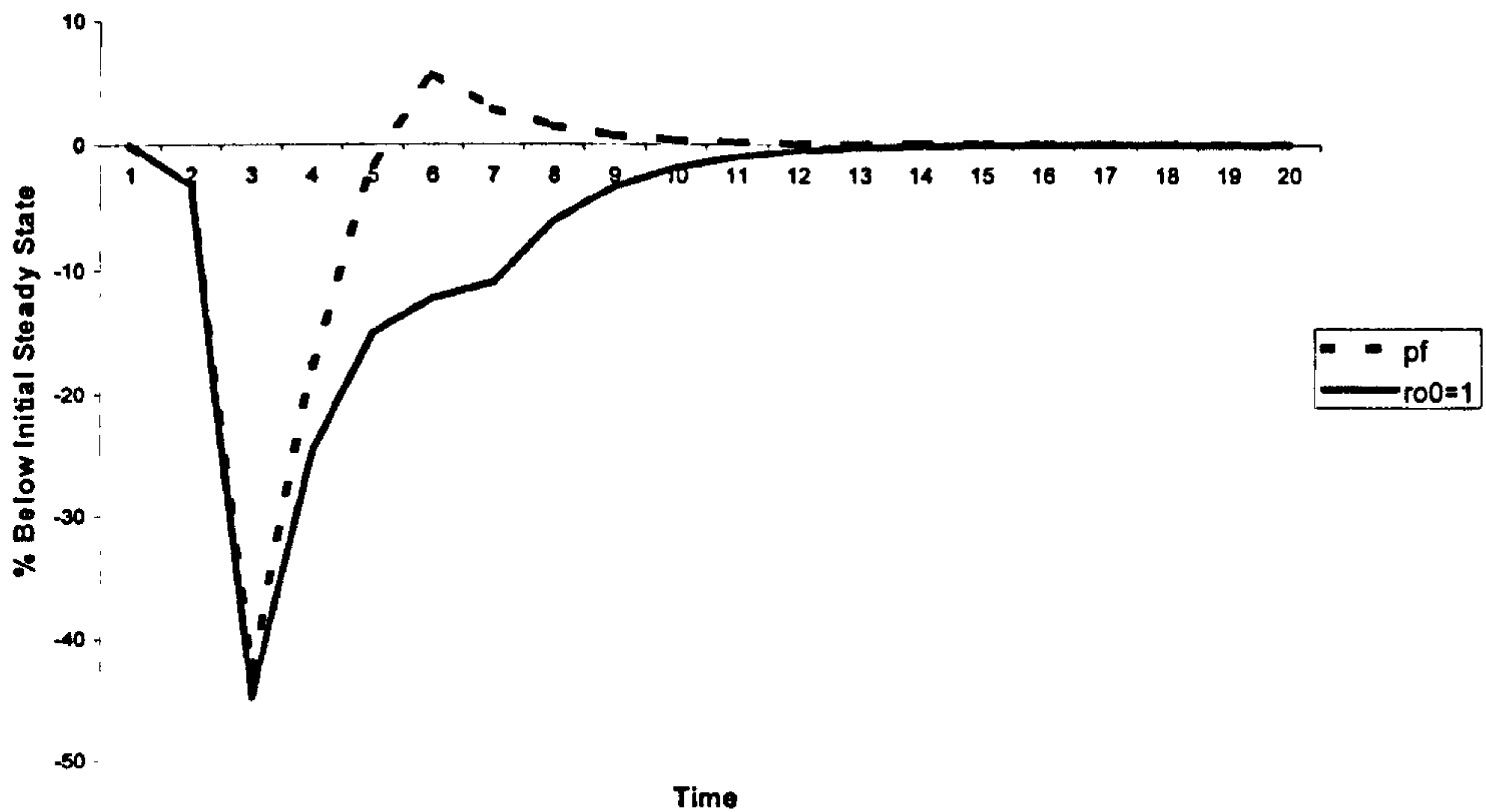


Figure 6-11: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 200%. 3 Years to Perfect Foresight. R1

significantly less engaged and responsive, in other words, more 'lazy' or less inclined to consider the announcement of the monetary authority regarding the money supply or they worry that as the slope of the short-run Phillips curve flattens, the monetary authority may be tempted to renege.

This demonstrates that the appearance of the 'disinflationary booms' in an environment with imperfect credibility depends on the type of learning involved relative to the speed of disinflation, in other words depends on the speed of learning relative to the speed of disinflation.

### 6.3.1.2. Expectation Updating (4.4)

The second expectation updating rule employed is when agents expect the disinflation policy to stall, in other words, when they expect an inflation rate equal to that of the previous period. Under this rule, the path of output is expected to be much closer to that achieved for the case of perfect credibility in comparison with that achieved under the first rule. This is due to the fact that under the second rule, the expected money supply is closer to that announced by the monetary authority, in comparison with the case in which the expected money supply is calculated according to the first expectation updating rule. The reason for this is that under the second expectation updating, the

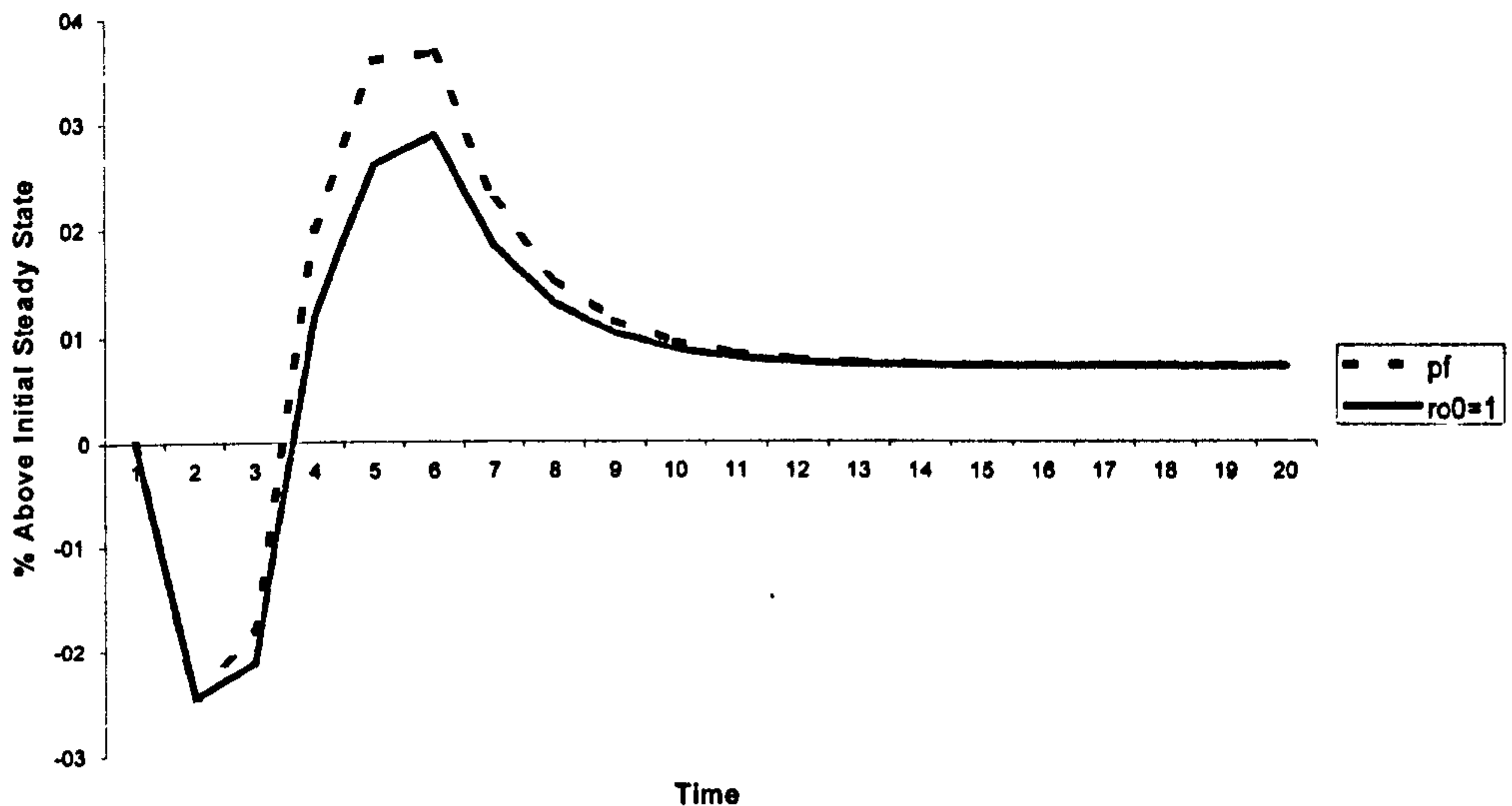


Figure 6-12: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight.  $R2$

level of the money supply that the private sector is thinking it is most likely to get, is much closer to that announced by the monetary authority in comparison with the first expectation updating. Under the second expectation updating the level of money supply expected by the private sector is that of the previous period, while under the first expectation updating the expected level is that of the moment of change in policy. This makes a major difference for the behaviour of output as the initial expectational errors are smaller under (4.4) than under (4.3).

In figure 6-12 it can be seen that under the second expectation updating rule, for concave learning, the output boom still appears, although it is smaller in comparison with the case of convex learning. This can be seen in figure 6-13.

This is explained by the fact under the second expectation updating, the expected money supply (calculated as the weighted mean between what the private sector thinks may be likely and the money supply announced by the authorities) is closer to the case of perfect foresight. From the analysis of the first expectation updating in section 6.2.1.1 it has been seen that the difference between the two types of learning arises from the way the private sector learns about the (level of the) money supply, and how fast it learns *at the beginning* of the change in policy. Learning can occur more quickly (under convex learning) or more slowly (under concave learning). If the expected level

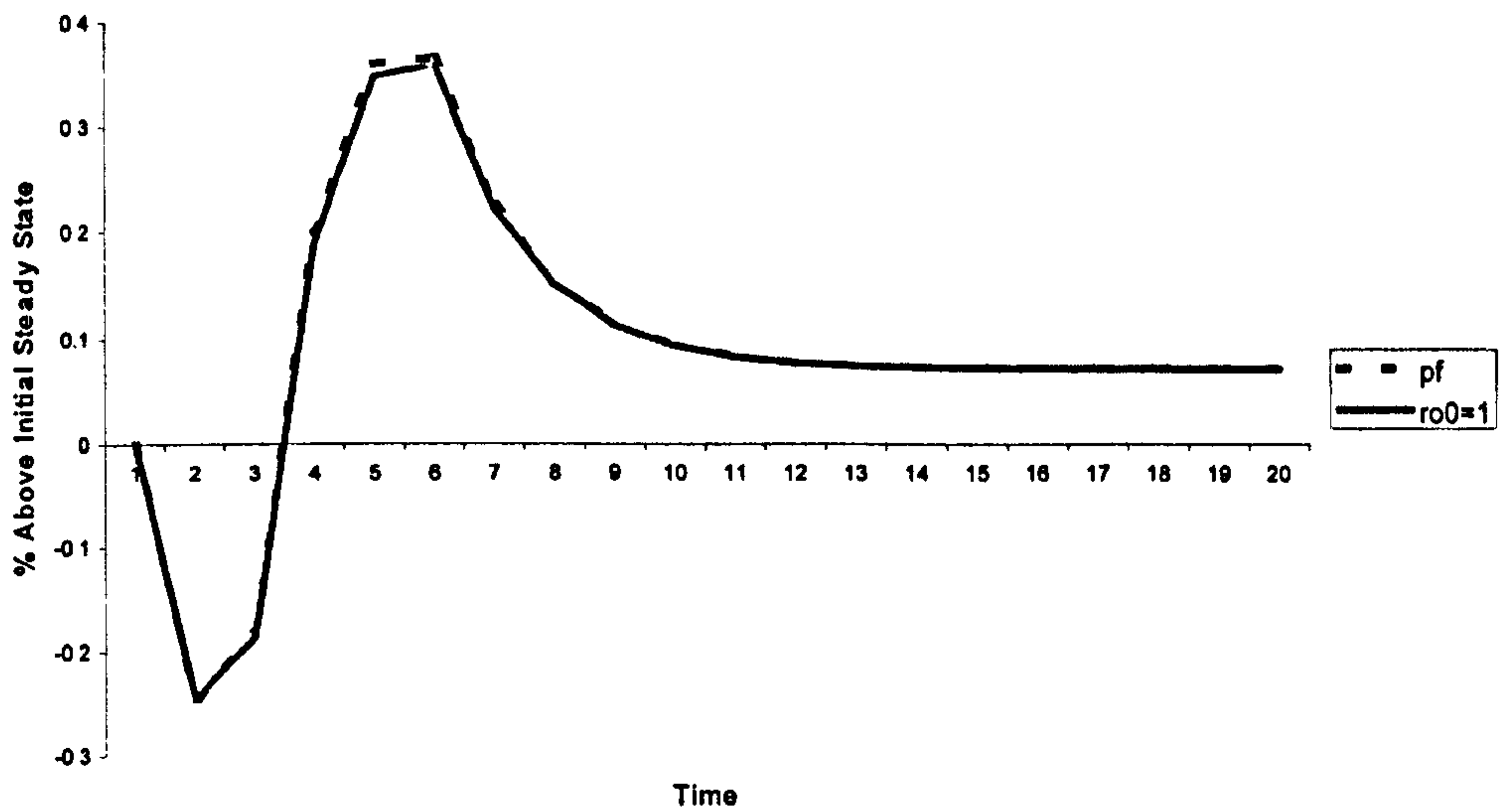


Figure 6-13: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight.  $R2$

of money supply under imperfect credibility is not much different from that achieved under perfect foresight (due to expectation updating), the type of learning does not have much room for manoeuvre. This is due to the fact that *at the beginning* of the change in policy, the initial expectational error under (4.4) (the difference between the expected money supply under the second expectation updating and that announced by the authorities - perfect foresight) is smaller than under (4.3). In this case, the type of learning does not have a big effect on the behaviour of output, as opposed to the first expectation updating rule, where the expected money supply is much different from that announced by the central bank. This leads the price setters, although they find themselves under imperfect credibility, to set prices much closer to those under the case of perfect foresight. Therefore, the output path under convex learning is extremely close to that of perfect foresight.

Comparing the output effects of concave and convex learning under the second expectation updating, it can be seen that many of the same qualitative results found under concave learning are present in convex learning, for both immediate and gradual disinflation of a small and big initial annual inflation rate. The difference between them is that under concave learning the output rises less than under convex learning (where it is very close to the output achieved under perfect foresight). This

suggests that the manner in which learning takes place makes a difference to output. This difference however, is diminished by the expectation updating type. If the private sector is willing to believe the money supply announced by the monetary authority at the beginning of the change in policy, output rises more than if they are reluctant to accept their announcement at the beginning, even if they are becoming more trusting as time goes by.

Also, under this second rule, the disinflationary boom still appears, under both concave and convex types of learning. The conclusion is that in an environment with imperfect credibility, the appearance of the 'disinflationary booms' also depends on the inflation the private sector thinks it is most likely to experience.

### 6.3.1.3. Conclusions

To summarize, under the *first expectation updating*, it has been shown that disinflation under concave type of learning is more costly in comparison with convex learning, as agents initially adjust only slowly towards the new announced path of the money supply. Under the convex path agents avoid some of the more costly mistakes early on in the disinflation common with concave learning. Therefore, the way the private sector learns at the beginning of the change in policy plays a major role. Furthermore, a boom in output occurs even under the case of imperfect credibility when there is concave learning, but this is no longer the case for convex learning.

Under the *second expectation updating* it has been seen first that the disinflationary boom appears, under both concave and convex types of learning. As expected, under concave learning output rises less than under convex learning.

Comparing the two types of non-linear learning under each of the expectation updating (4.3) and (4.4) it becomes apparent that the speed at which the private sector learns at the beginning of the change in policy is essential under both rules. However, the output effects of the private sector's learning are diminished under the second expectation updating rule. Another aspect that can be observed from here is that the inflation the private sector thinks is most likely to get has a stronger output effect than the type of learning involved.

Regarding the 'disinflationary booms' it can be said that these may or may not disappear in an environment with imperfect credibility, depending on the speed of

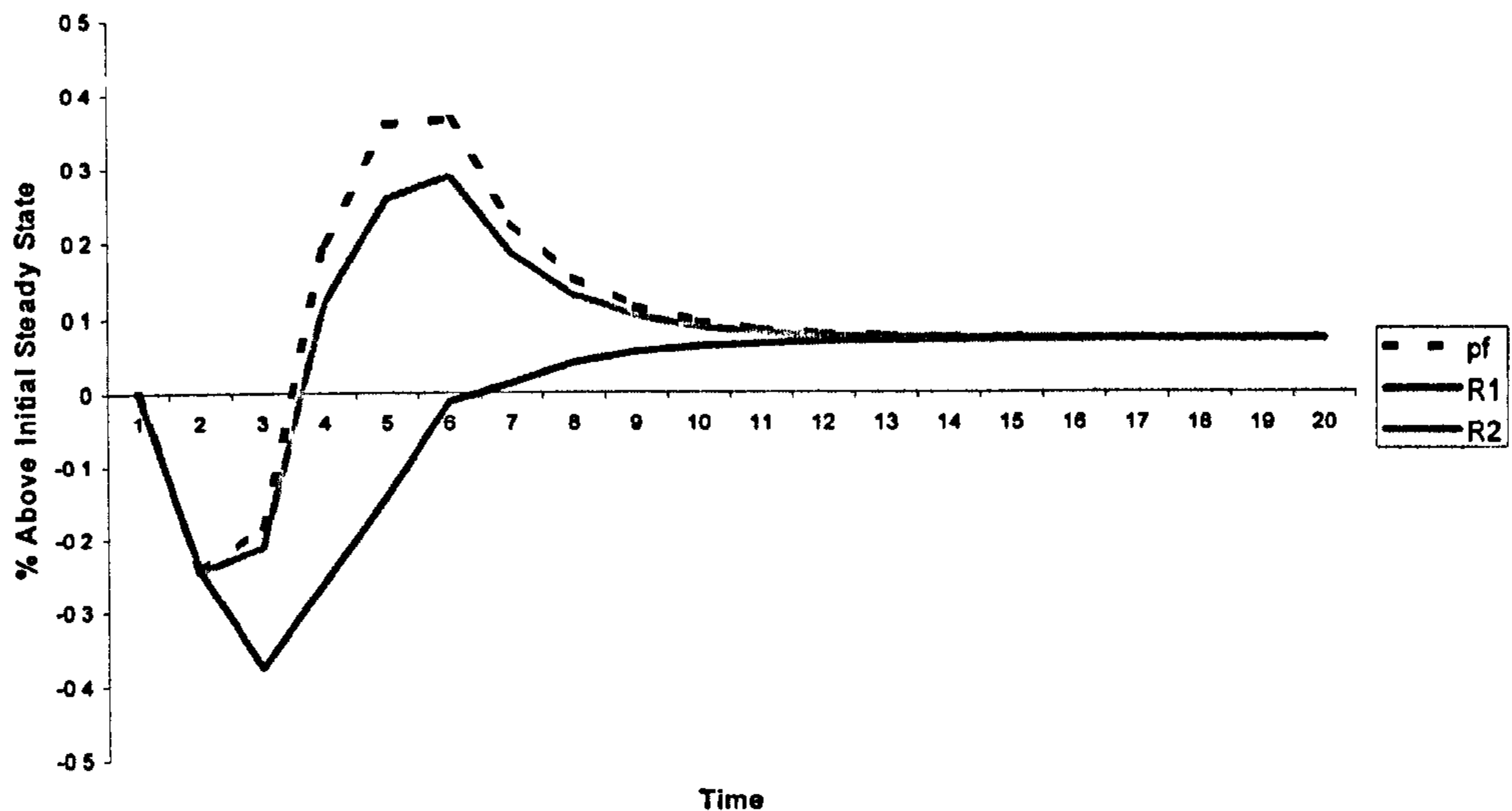


Figure 6-14: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1. R2

learning relative to the speed of disinflation<sup>3</sup>. If under the first expectation updating rule the question was whether the output boom will appear or not, under the second expectation updating rule the question is whether the output boom will be as prominent as under perfect foresight. In order to achieve disinflation with minimal output loss, the learning process should be speeded up at the beginning of the disinflation process.

### 6.3.2 The Output Effect of the Expectation Updating

Given the results of the previous section, the output effects of the expectation updating are noticeable. Depending on the way expectations are updated, the output effects of the type of learning can be affected. As presented in section 6.3.1.2, it can be seen that these are diminished under the second expectation updating. The cost of disinflation encountered under concave learning is reduced under the second rule, since the initial expectational errors are much reduced. For this reason, under the second rule the concave paths of output are very close to those of perfect foresight.

Figure 6-14 plots the output effects of a gradual disinflation ( $T = 6$ ) of a 3%

<sup>3</sup>The robustness of these results holds for the various values of each of the other two factors determining the learning process: the initial level of the lack of credibility and the length of the learning period over which perfect foresight is attained. This will be further seen in the following sections.

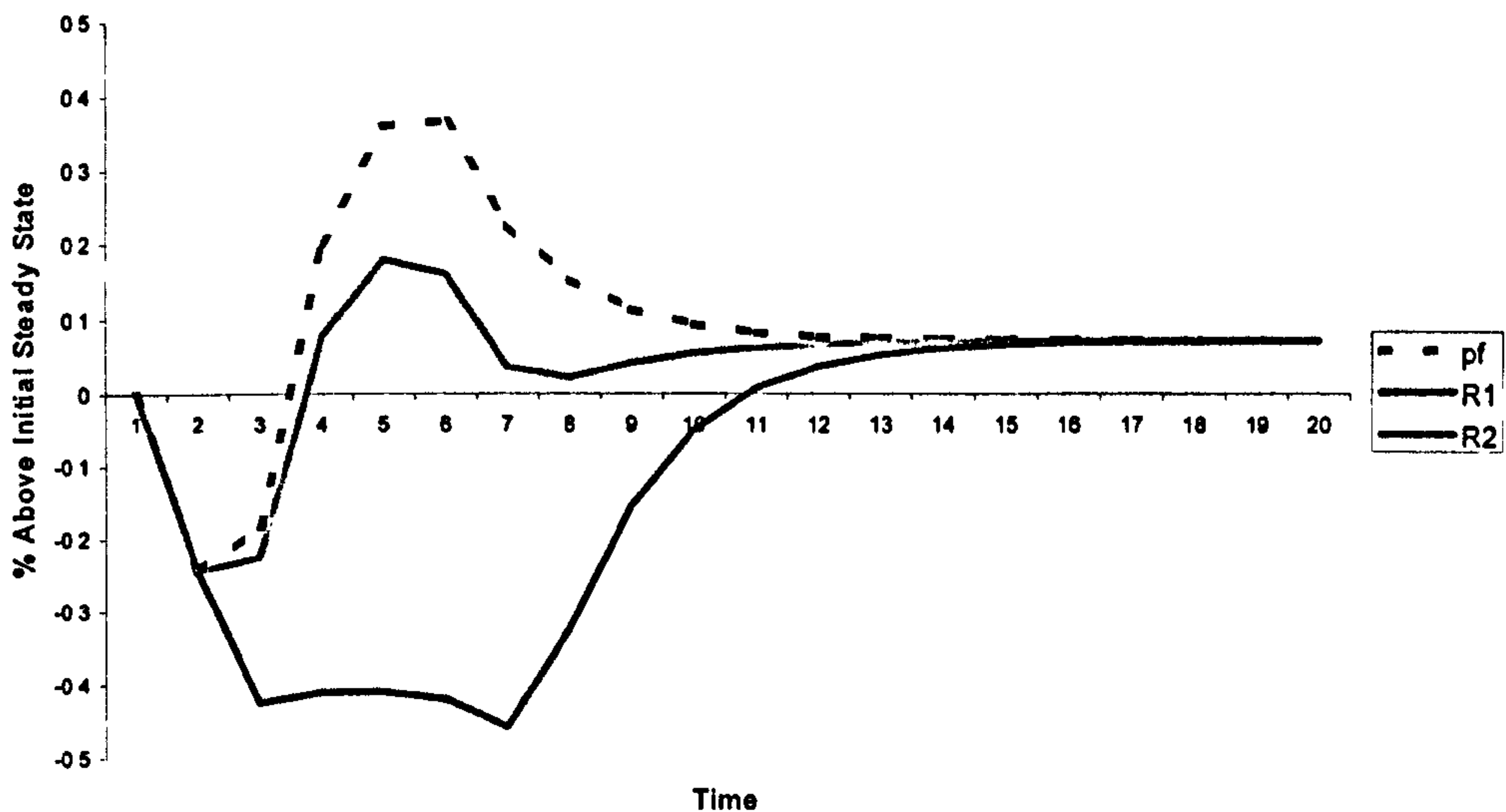


Figure 6-15: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R1. R2

initial annual inflation rate, for  $N = 6$ , under concave learning, for both expectation updating studied here. Under the first rule recession appears, while under the second rule a disinflationary boom is registered. These results are better pictured in figures 6-15 and 6-16 where the output effect of the expectation updating is more enhanced by an increase in  $N$ . Under the first rule, the recession becomes longer and more severe as  $N$  increases, while under the second rule the output boom is reduced, registering a cyclical behaviour.

This difference in output effect which is due to the reasons mentioned earlier, is minimized under convex learning, as seen from figures 6-17 to 6-19, even though the length of the learning period over which perfect foresight is achieved is increased.

Under either concave or convex learning, and especially under the former, the output gain can be improved if the private sector think that the inflation rate that will occur is that of the last period rather than that at the moment of the change in policy. Any level of inflation the private sector thinks is most likely to get other than that at the moment of the change in policy is preferable with respect to the output gain.

To summarize, it can be said that depending on the expectation updating, the output effects of the type of learning are minimized, especially under the second expectation updating. These are reduced under concave learning and especially under

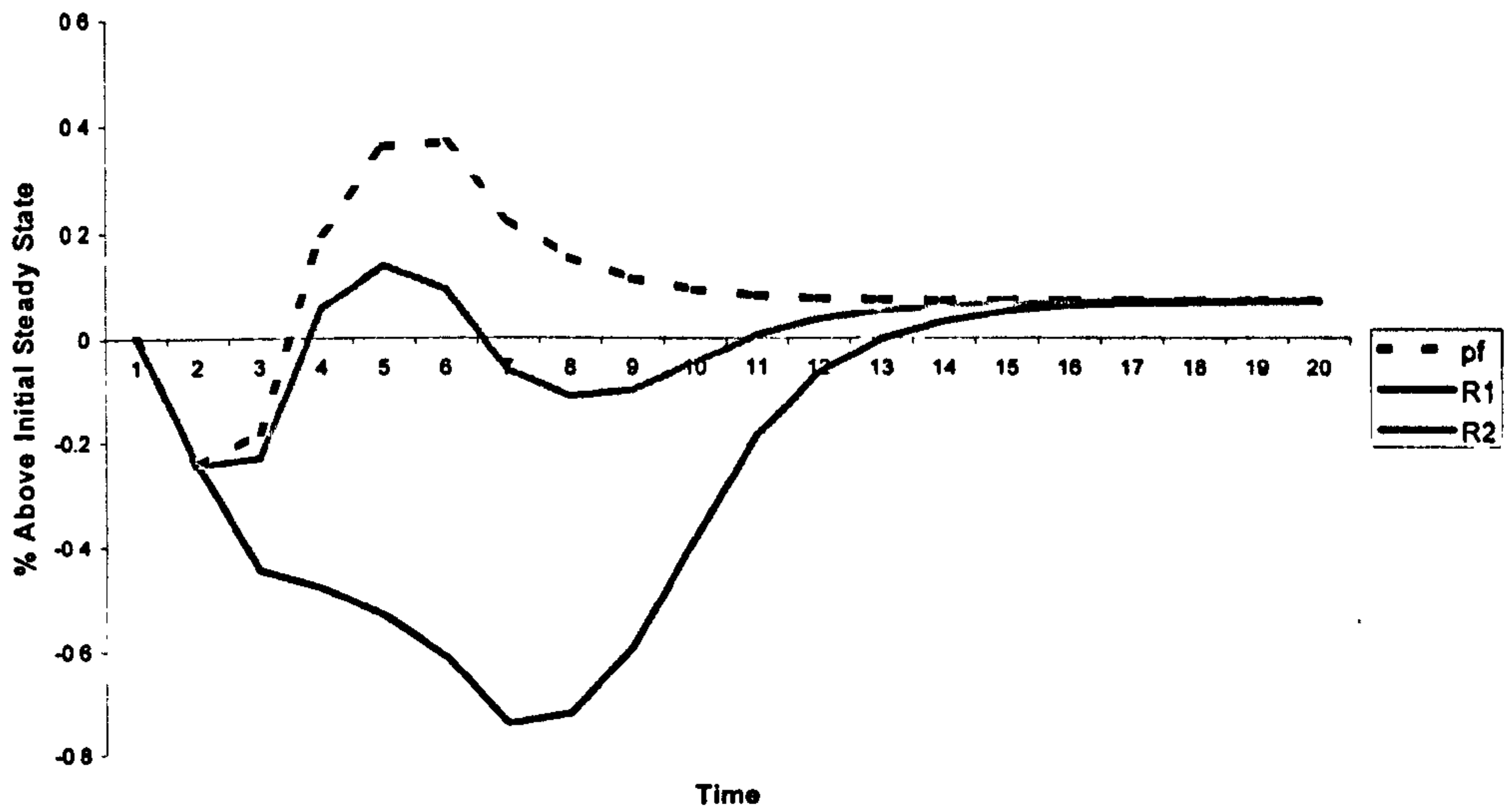


Figure 6-16: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R1. R2

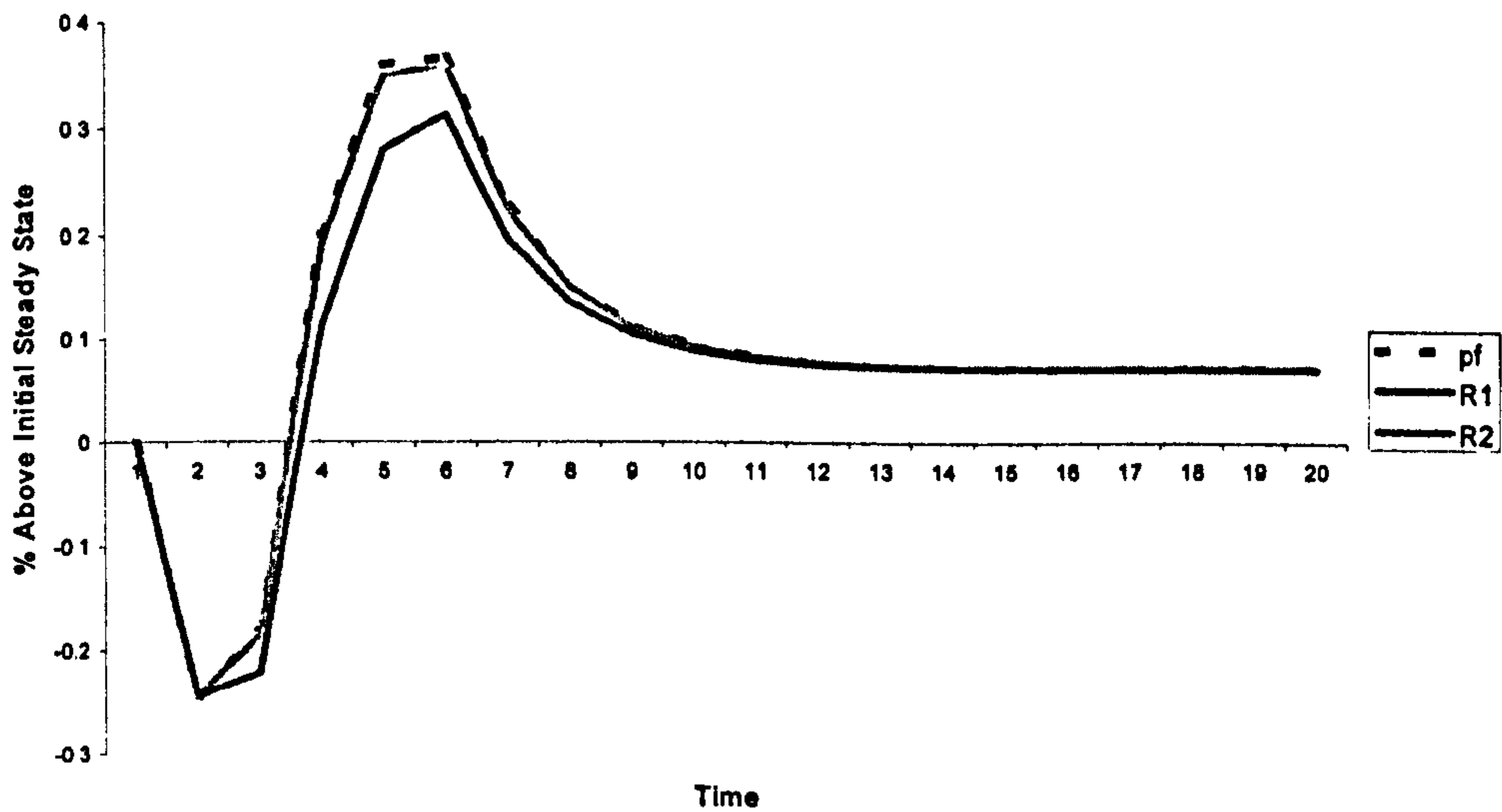


Figure 6-17: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1. R2



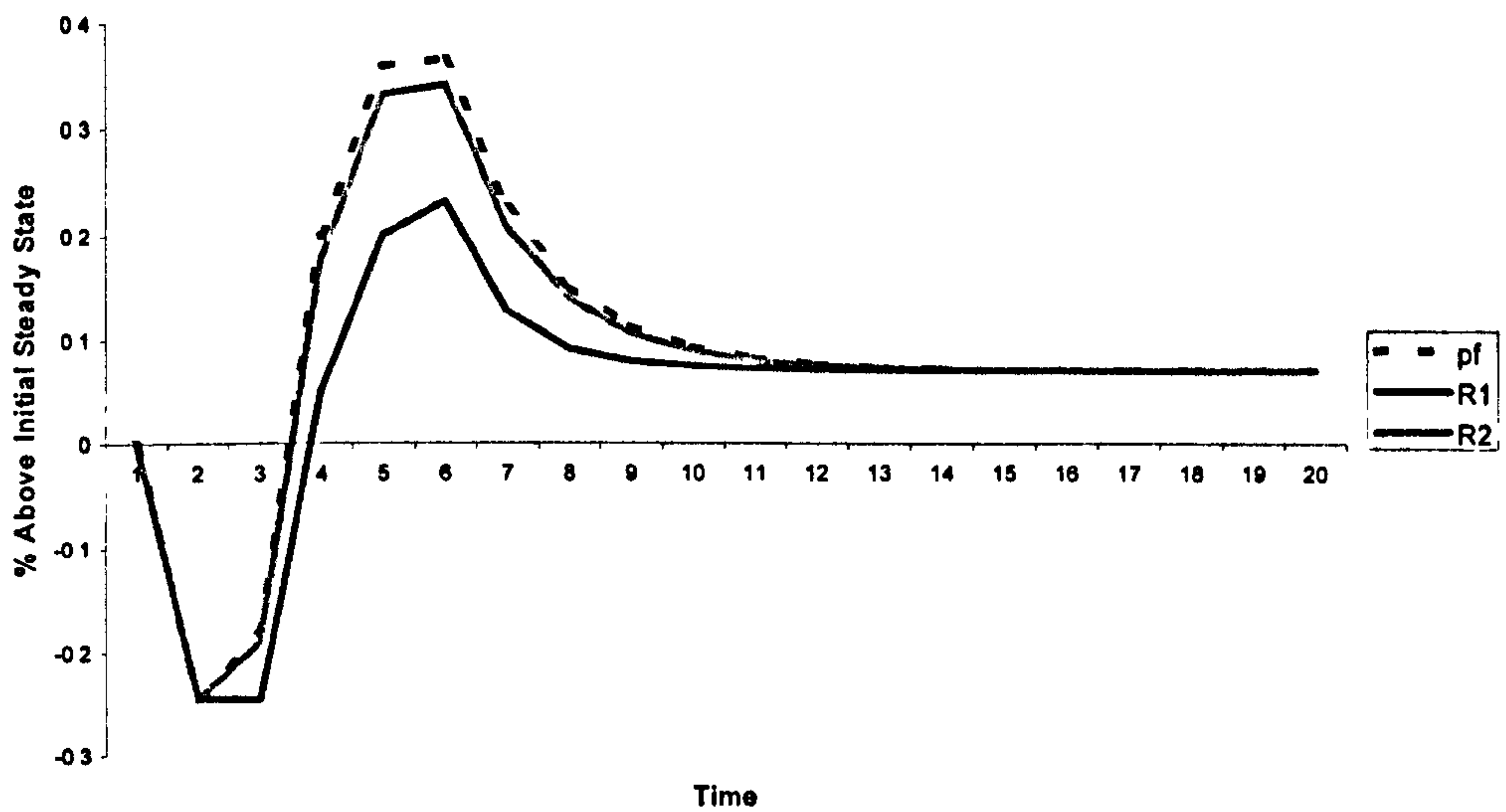


Figure 6-18: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R1. R2

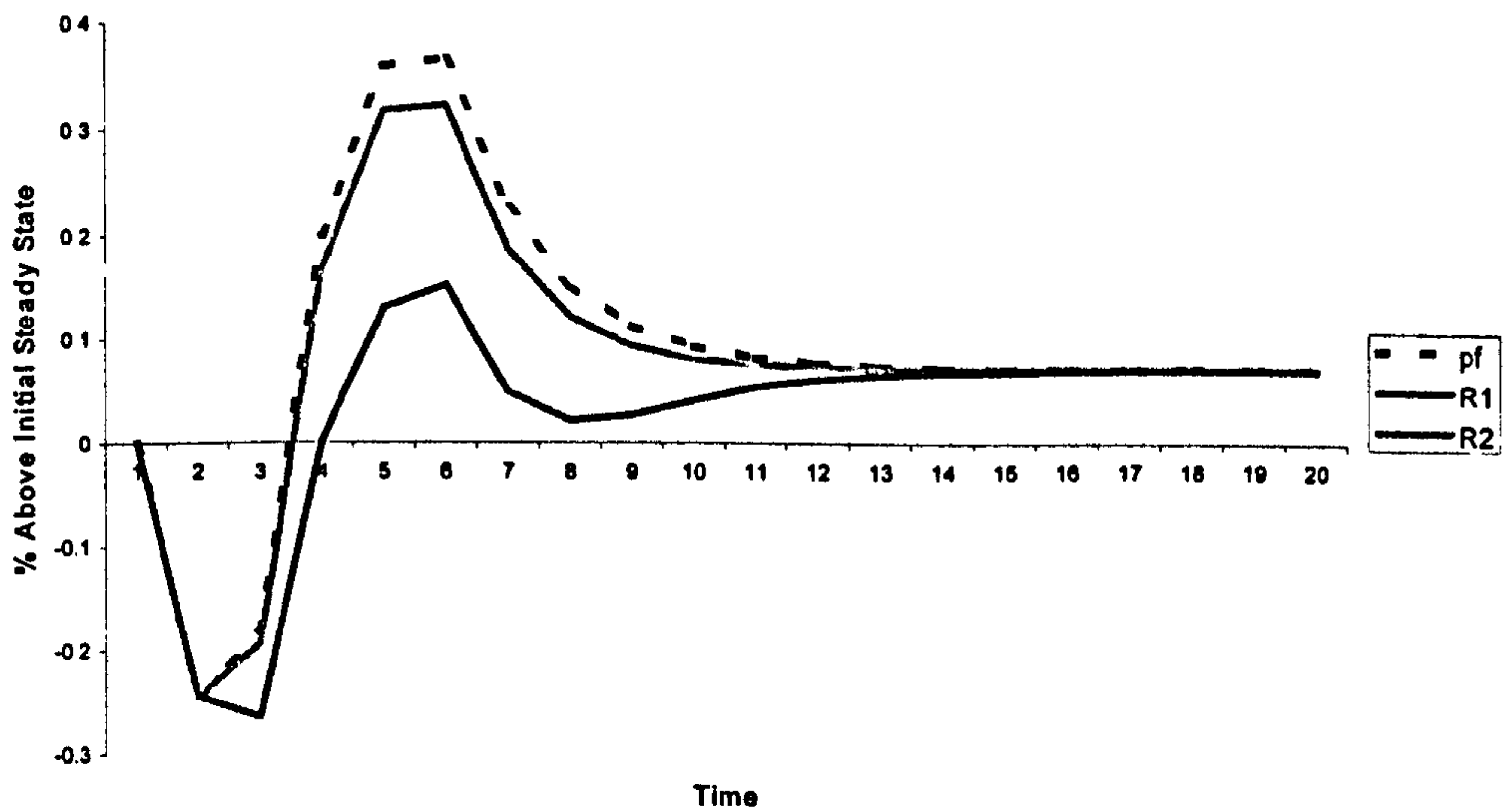


Figure 6-19: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R1. R2

the second rule as the initial expectational errors are also reduced. For this reason, under the second rule, the concave paths of output are extremely close to those of perfect foresight. Moreover, in the light of these results, deflationary booms are more likely to occur under (4.4) and, as before, concave updating tends to make these booms disappear.

### 6.3.3 The Output Effect of the Length of the Learning Period

The aim of this section is to identify the output effect of the length of the learning period over which perfect foresight is achieved during a disinflationary process. The length of the learning process (the period of time over which  $\rho_0$  starting from 1 gets to 0) is not easy to ignore.

A comparison with the case in which learning takes place over 3 years ( $N = 6$ ) to the one in which perfect foresight is achieved over 4 and 5 years (corresponding to  $N = 8$  and  $N = 10$ ), suggests that under either concave or convex learning and regardless of the expectation updating, the longer the period over which learning is achieved, the longer and more severer the recession will be.

To underline the output effect of the length of the learning period the case in which perfect foresight is achieved over a period of 3 years is compared to the cases in which perfect foresight is achieved over 4 and 5 years. The study is carried out for the same case in which the annual initial inflation rate of 3% is eradicated over a period of 3 years, under concave learning. Moreover, we follow the pattern from section 6.2.1, presenting first the behaviour of output under rule (4.3) for both learning types, and then under rule (4.4).

#### 6.3.3.1. Expectation Updating (4.3)

Under *concave learning* when comparing figure 6-6 to figure 6-20, the effect one more year of learning has on output is striking. The recession which appears under the case of a 3 year learning period ( $N = 6$ ) becomes more severe and lasts for longer for the case in which the learning process is prolonged by one year ( $N = 8$ ).

This is explained by the fact that the speed of learning which is affected by the length of the learning period over which perfect foresight is attained ( $N$ ), is decreasing and therefore it takes longer for the private sector to believe the government's

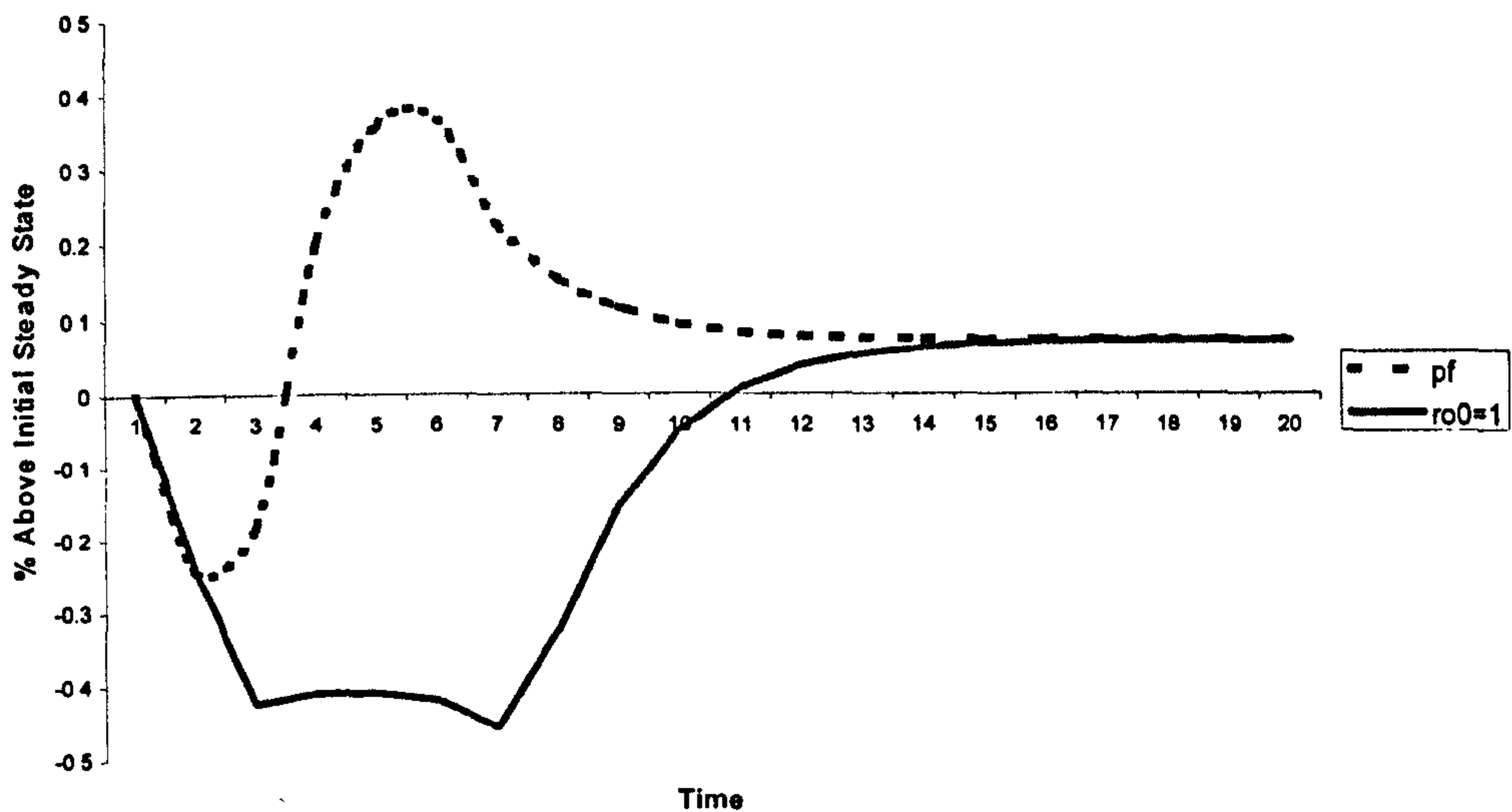


Figure 6-20: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R1

announcement. In this case, one more year is needed to achieve perfect foresight and leads to a recession that lasts for two years more before output returns to the initial steady state.

Figure 6-21 in which the learning process is considered to take place over 5 years ( $N = 10$ ), shows that the output falls further in the subsequent periods (almost to 0.75 below the initial level of steady state compared to 0.45 under the case in which the learning take place over 4 years ( $N = 8$ )), and stays under the initial level of steady state achieved under perfect foresight even longer. Therefore, under the case of concave learning, the length of time over which the private sector's learning process takes place is reflected in the length of the recession period. The longer the learning period of the private sector, the longer and more severe the recession period will be. It can also be said that the slower the speed of learning the longer the recession will be and also the higher the cost of disinflation will be. After the private sector achieves perfect foresight, it still takes another year for output to return to the initial steady state and on average two more years to achieve the level of perfect foresight steady-state output.

The conclusion is that the shorter the learning process (in this case no longer than 3 years), the better the output gain and the smaller the cost of disinflation. It can be stated that under a gradual disinflation and concave learning, a long learning process

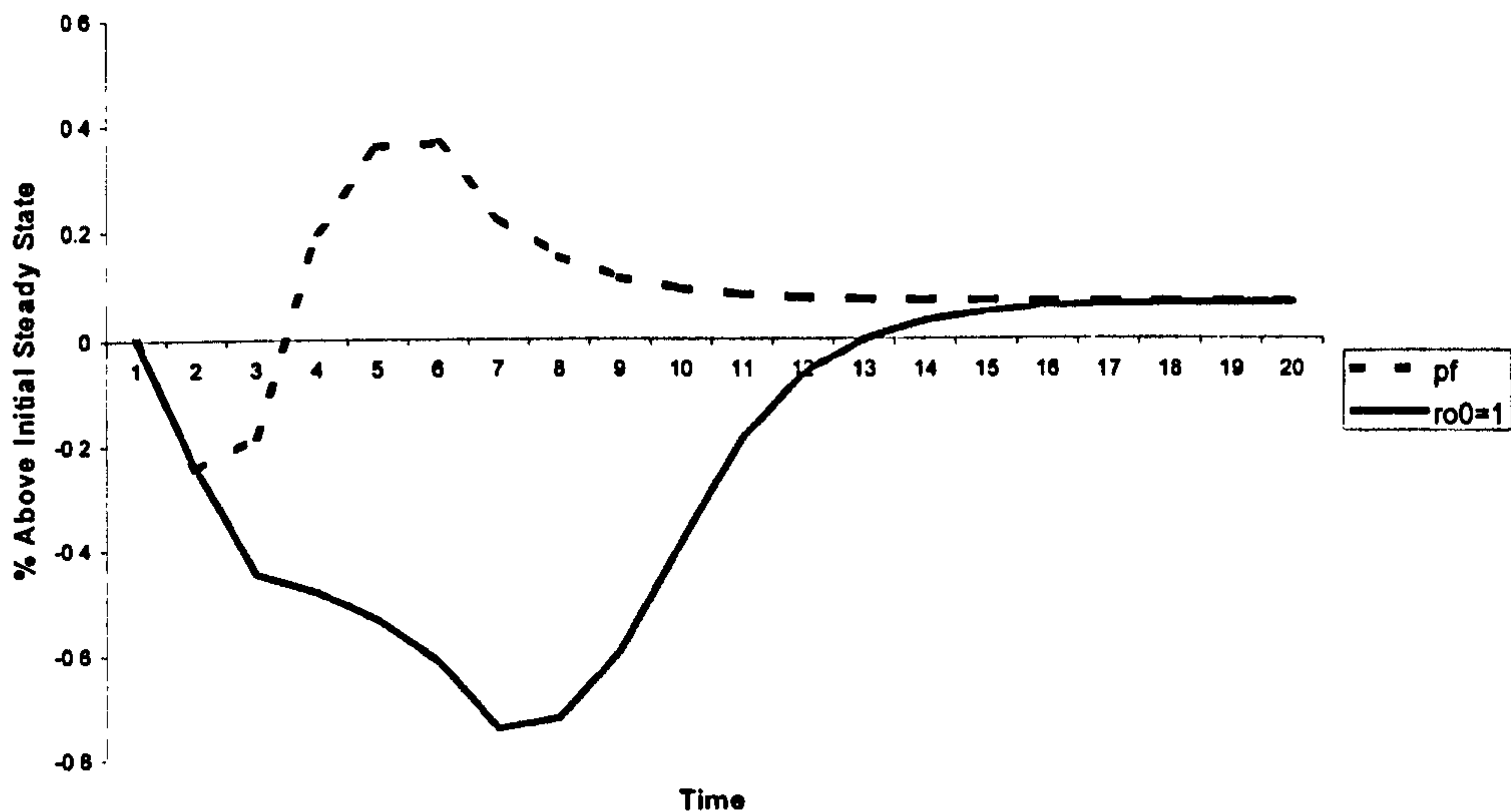


Figure 6-21: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R1

only has negative effects on the output prolonging the recession. If the learning process is long, the recession persists over a longer period of time and is more severe.

Under the case of *convex learning* a disinflationary boom occurs, as seen in figure 6-22, plotting the case of  $T = 6$ , and  $N = 6$ . Figure 6-23 plots the case in which 4 years are necessary to achieve perfect foresight ( $N = 8$ ). Here the disinflationary boom is reduced and so also is the output gain.

This result is more obvious when comparing the case in which the learning process takes place over 3 years to the case in which the learning process takes place over 5 years ( $N = 10$ ), as plotted in figure 6-24. In this latter case, the output boom is much smaller. It is almost 0.15% above the initial steady state compared to 0.225% above the initial steady state for 4 years. Moreover, in comparison with the other two cases in which learning takes place over 3 and 4 years ( $N = 6$ ,  $N = 8$ ), for the case in which learning takes 5 years ( $N = 10$ ), output falls below the perfect foresight level of steady state before returning to it.

The **conclusion** is similar to that achieved by studying the effect of the length of the learning process under concave learning. While gradually disinflating a small initial annual inflation rate under either concave or convex learning, the length of the learning process is directly reflected in the length of the recession. Of course, this is

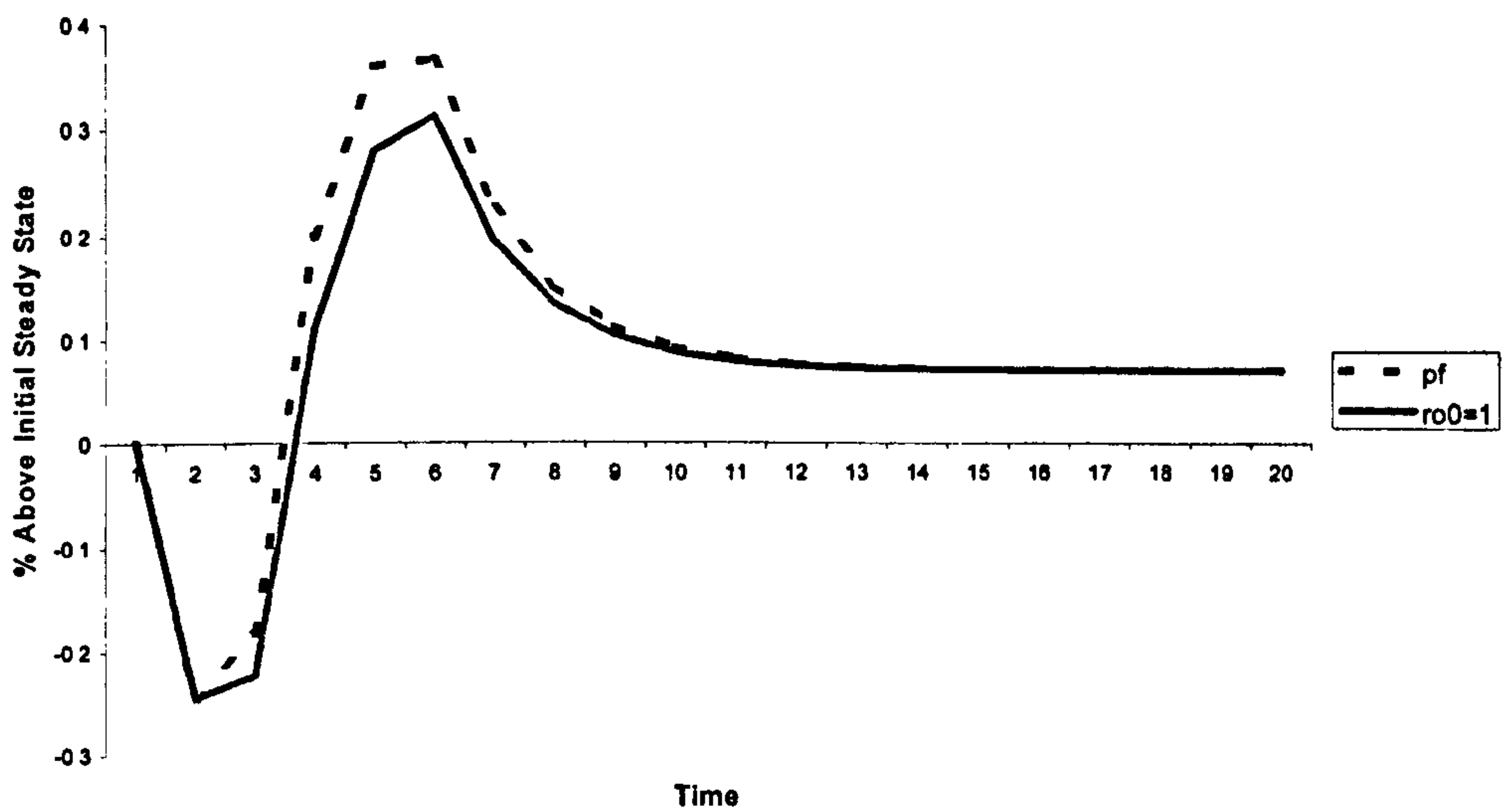


Figure 6-22: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1

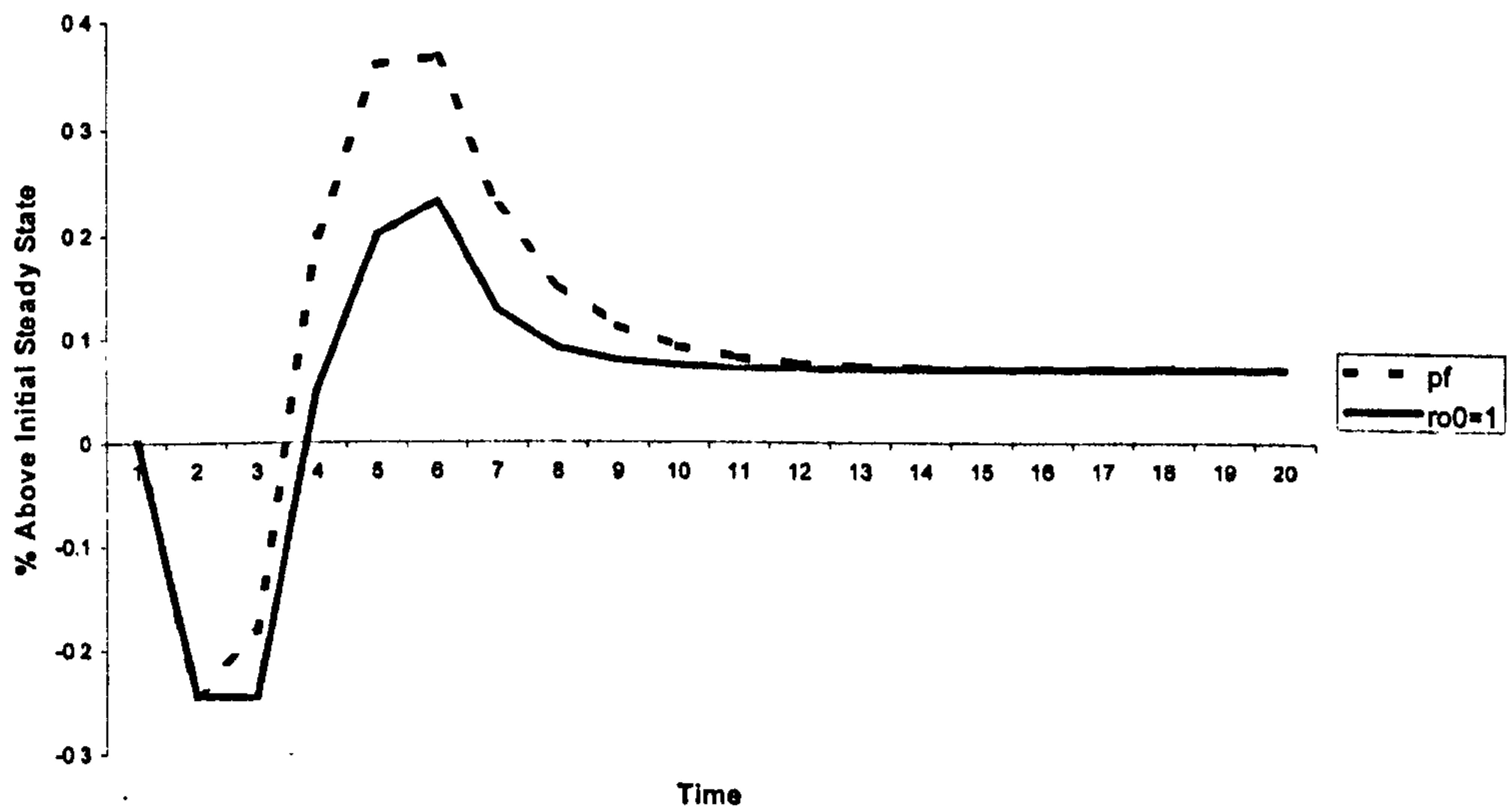


Figure 6-23: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R1

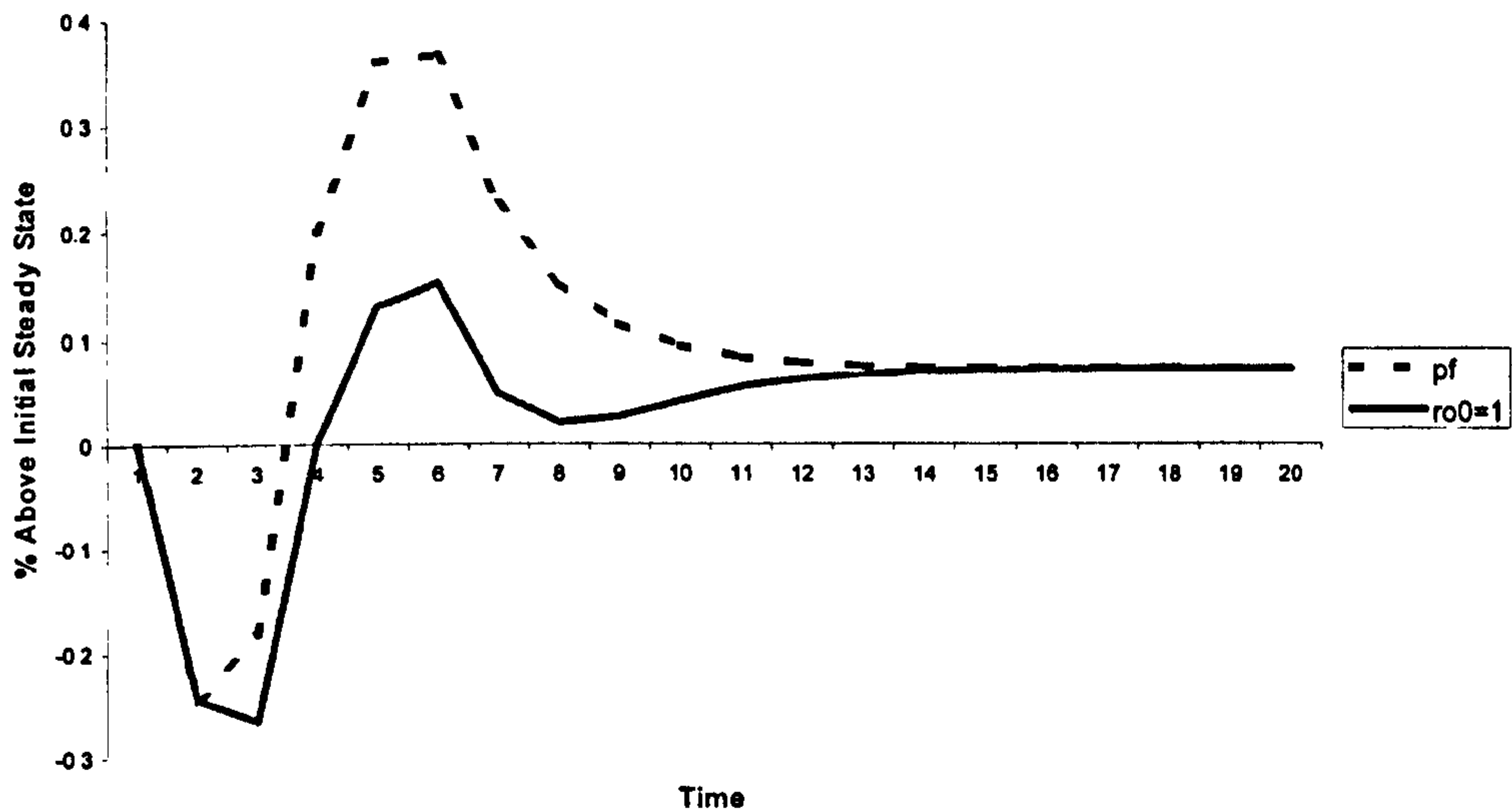


Figure 6-24: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R1

more severe under concave learning.

### 6.3.3.2. Expectation Updating (4.4)

As seen in figure 6-12, in the case of *concave learning* and under the assumption that the learning process takes 3 years ( $N = 6$ ), an output boom appears. However, this output boom is diminished and the output even falls under the perfect foresight steady state level when the learning process is one year longer ( $N = 8$ ), as plotted in figure 6-25. Under the case of concave learning, when the private sector is quite reluctant to believe the government announcement at the beginning of the change in policy, the output falls before getting to the perfect foresight steady state level. Although the (negative) output effects of the public being reluctant to believe the government's announcement at the beginning of the regime change are minimized under the second expectation updating, they occur in the future due to an overlapping effect from price setting behaviour. Hence, after the disinflationary boom, output begins to fall before ending at the perfect foresight steady state level.

Looking at figure 6-26 which plots the output under the assumption that the learning process takes 5 years ( $N = 10$ ), the overlapping effect is enhanced. In this case the output has a cyclical behaviour. After falling initially (being for two consecutive

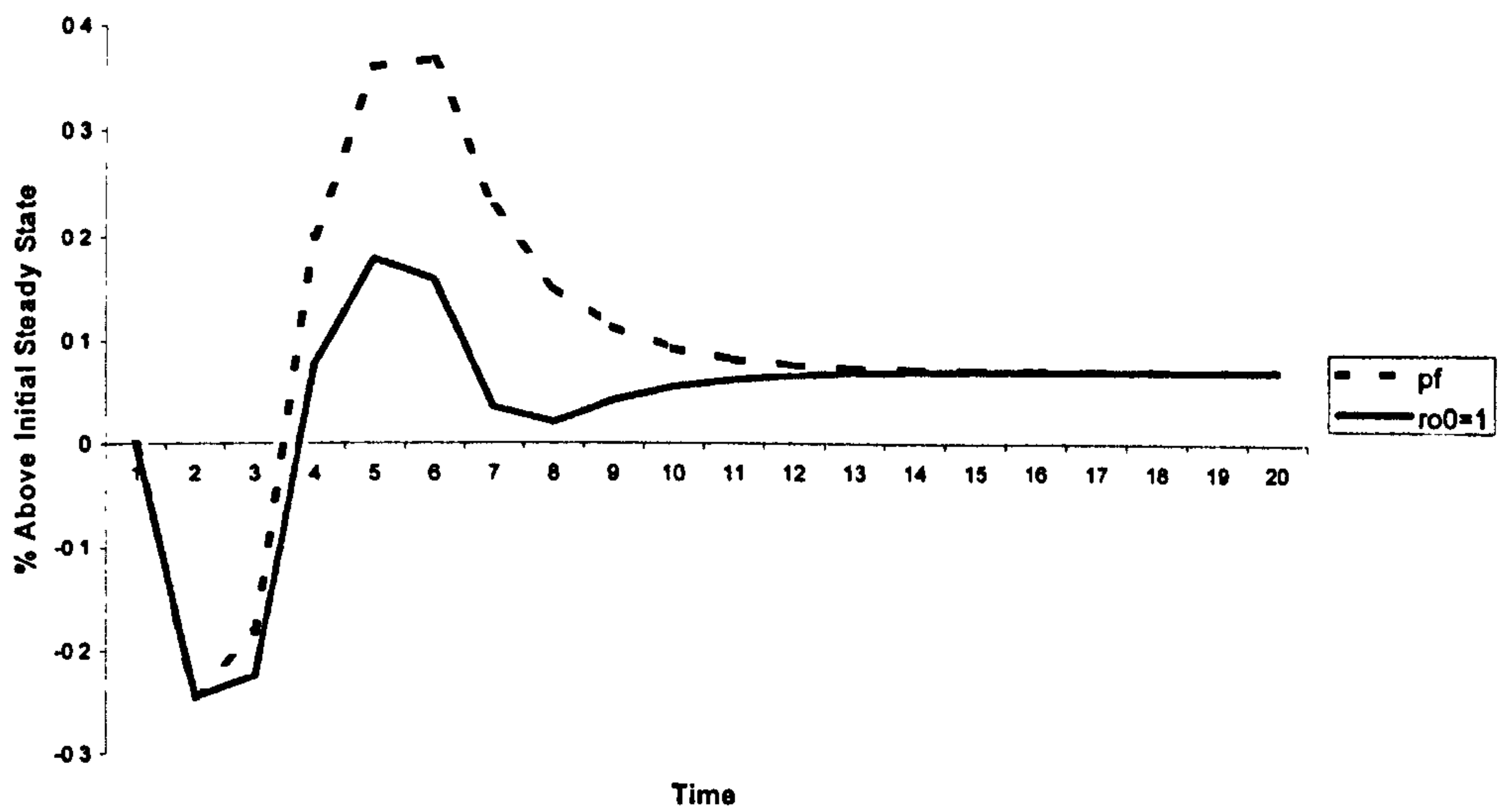


Figure 6-25: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R2

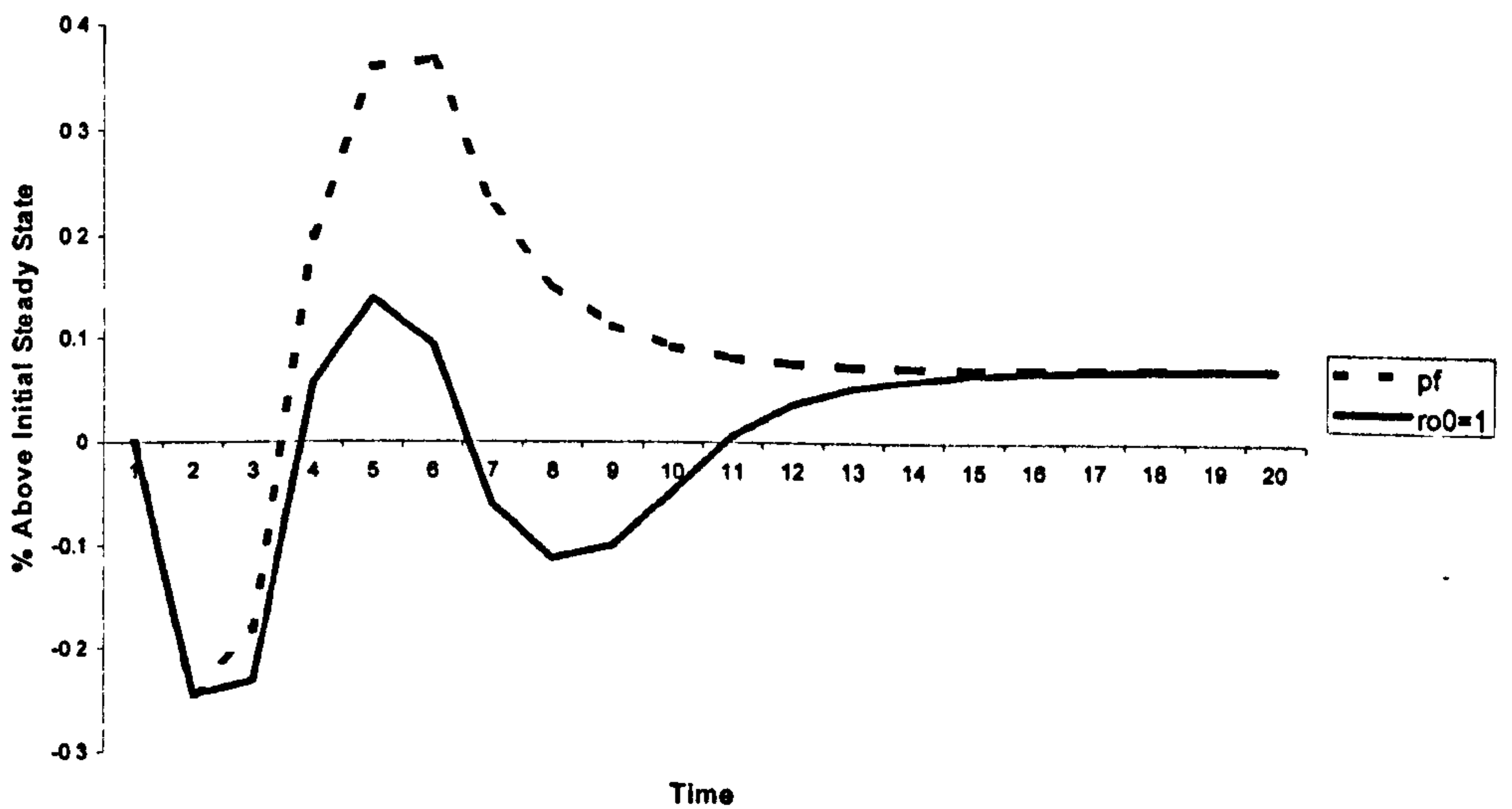


Figure 6-26: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R2

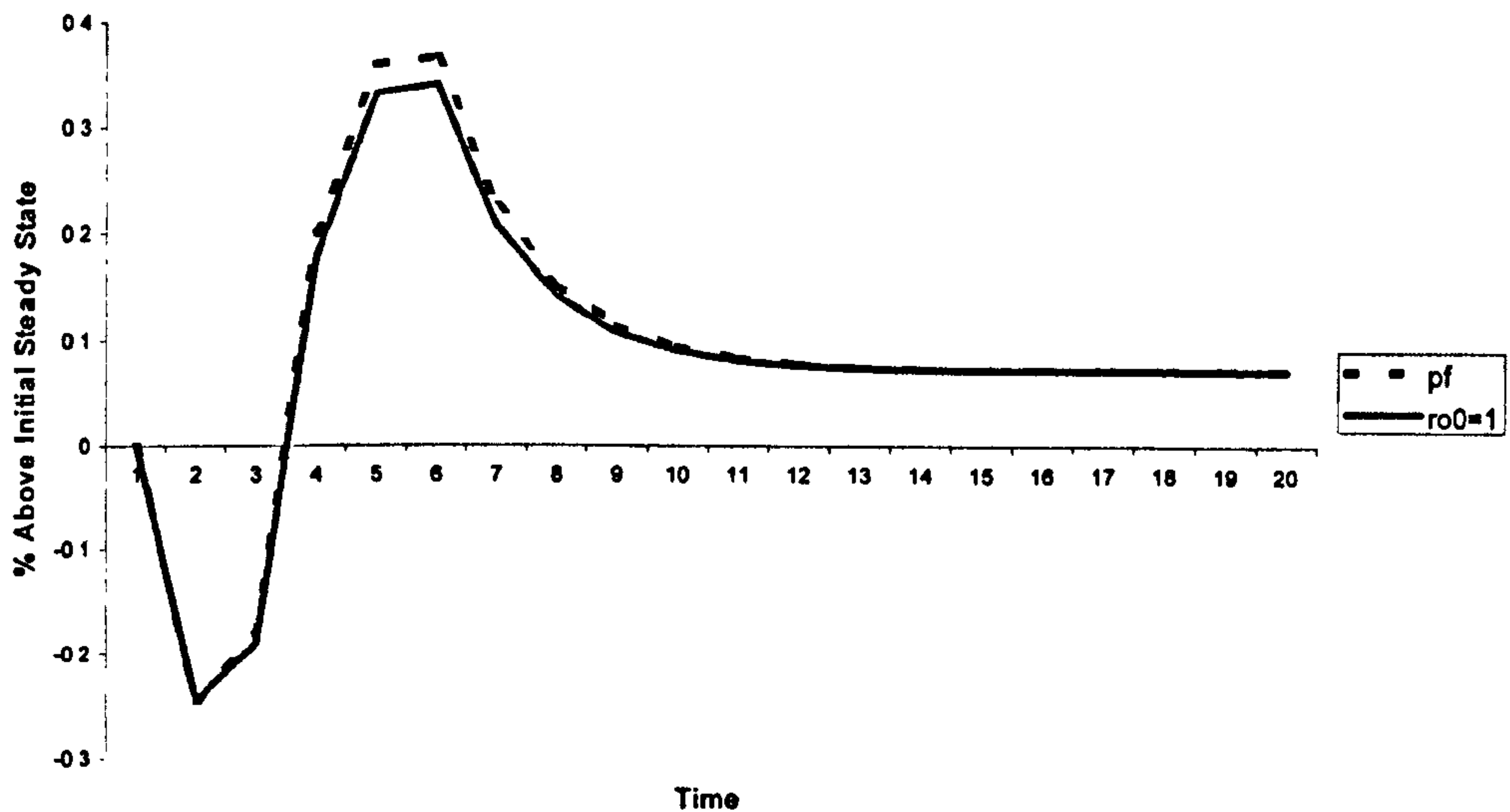


Figure 6-27: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 4 Years to Perfect Foresight. R2

periods of time in recession), output records a much smaller (only 0.14% above the initial steady state comparing to 0.19% above the initial steady state for  $N = 8$ ) disinflationary boom, followed again by recession. A longer learning process induces a cyclical effect on the output's behaviour, although the amplitude is reduced with each cycle. This reduction is explained by the fact that as the learning process takes place, the private sector gets closer to perfect foresight. At the same time, output is lead towards its perfect foresight steady state level.

The robustness of the conclusion holds: the length of the learning process is directly reflected in the size of the output loss or gain as the same results are obtained for higher initial annual inflation rates, i.e. 10%<sup>4</sup>.

For *convex type of learning*, as seen from figures 6-13, 6-27,6-28, the output path is very close to that of perfect foresight, due to the reasons explained earlier. The output boom appears, and in comparison with the case of perfect foresight is diminished only under bigger values of  $N$ , but not by much. The conclusion from the previous subsection, therefore, is valid for convex learning.

<sup>4</sup>The presentation of this case has not been included in the thesis.



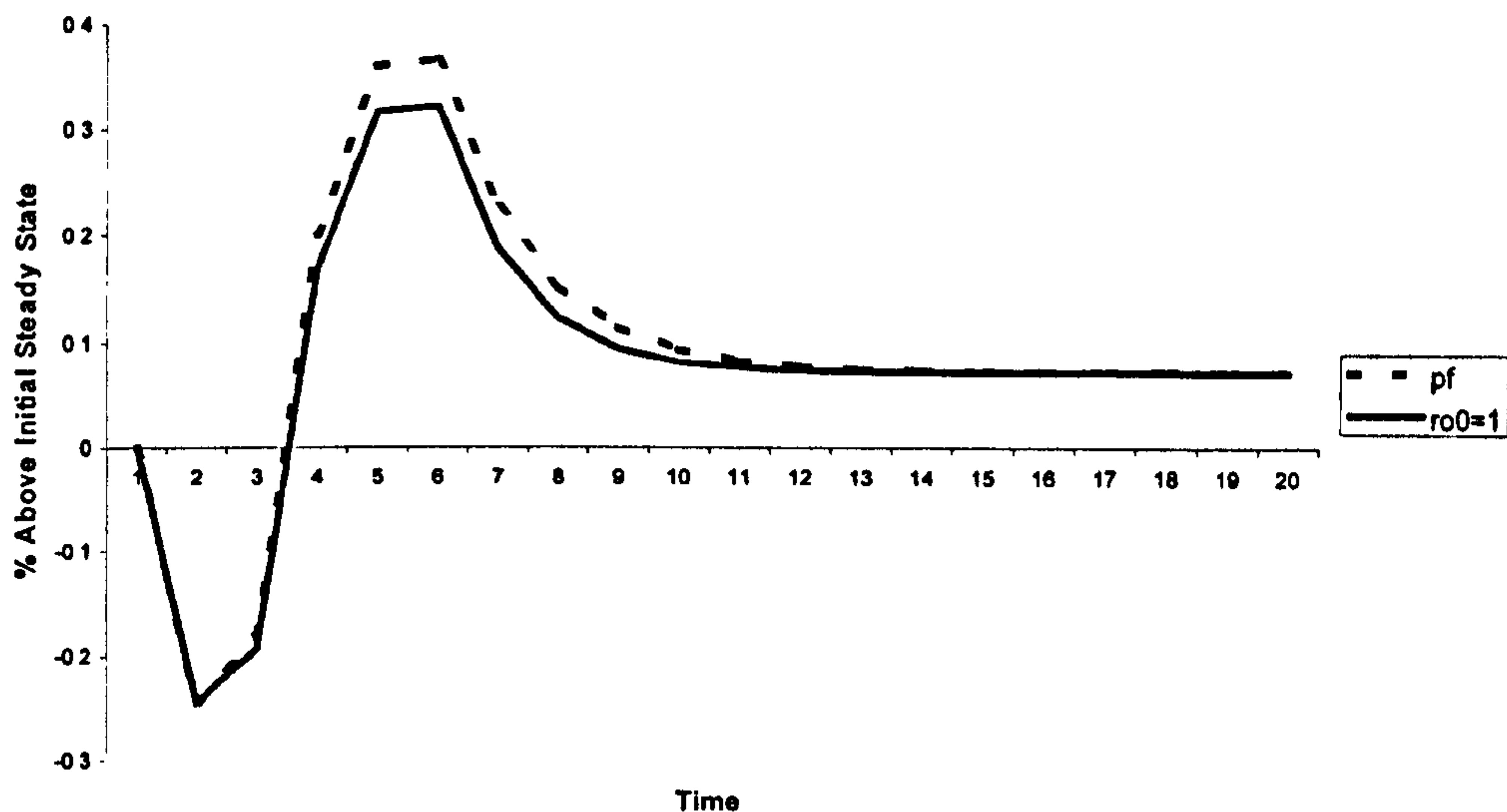


Figure 6-28: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 5 Years to Perfect Foresight. R2

### 6.3.3.3 Conclusions

To summarize on the output effects of the length of time over which learning takes place during a disinflationary process, it can be said that disinflating gradually, under either concave or convex learning and regardless of the expectation updating, the longer the period of time over which learning is achieved, the longer and more severe the recession will be and also the higher the cost of disinflation will be. However, under concave learning, the output departures from the perfect foresight path are significant. These departures are increased by a longer learning period resulting in even higher output loss. For this reason the recession is more severe and the cost of disinflation higher. Nevertheless, the output loss appears to be minimized under the second expectation updating regardless of the length of the learning period, as described in the previous section. Second, although the (negative) output effects of the public being reluctant to believe the government's announcement at the beginning of the regime change are minimized under the second expectation updating, they occur in the future as an overlapping pricing effect occurs. This makes the output loss persist and appear later on into the future. Hence, for learning periods greater than 5 years and small inflation rates, output registers cyclical behaviour.

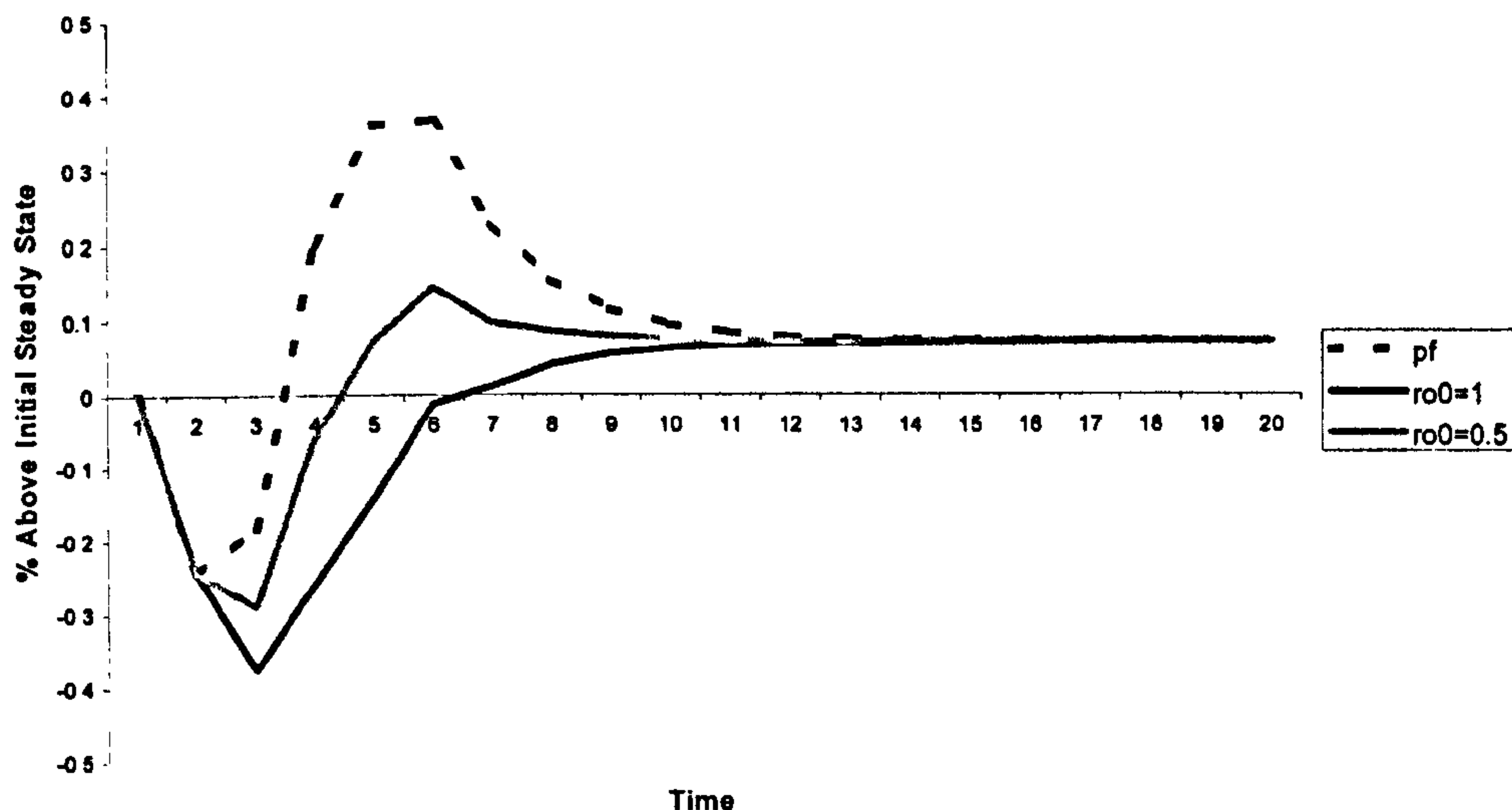


Figure 6-29: Output Effects of Gradual Disinflation under Concave Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1.  $\rho_0=1$ .  $\rho_0=0.5$

### 6.3.4 The Output Effect of the Initial Level of Lack of Credibility

An initial higher level of private sector trust in the monetary authority's announcement ( $\rho_0 < 1$ ), would make the disinflation less costly, in comparison with the case of absolute lack of credibility ( $\rho_0 = 1$ ). However, no studies have been carried out employing the measure of lack of credibility used in this study. Moreover, it appears to be difficult to quantify. This is the reason the entire study presented here has been carried out under the assumption that there is either a complete or an absolute lack of credibility. A number of experiments were run with different intermediate values for the initial level of lack of credibility,  $\rho_0 \in (0, 1)$ . In the context of this thesis it can not be said how to measure an intermediate half way initial level of credibility of the private sector in the government's announcement at the beginning of the change in policy except by  $\rho_0 = 0.5$ .

Figure 6-29, shows the output effects of such degree of lack of credibility for concave learning. Comparing the output for the case of  $\rho_0 = 0.5$  to the case of absolute lack of credibility ( $\rho_0 = 1$ ), it can be seen as expected, a shorter and less severe recession than under the case of absolute lack of credibility. Furthermore, the path of output in this case records an output boom. Over all, the output gain is bigger.

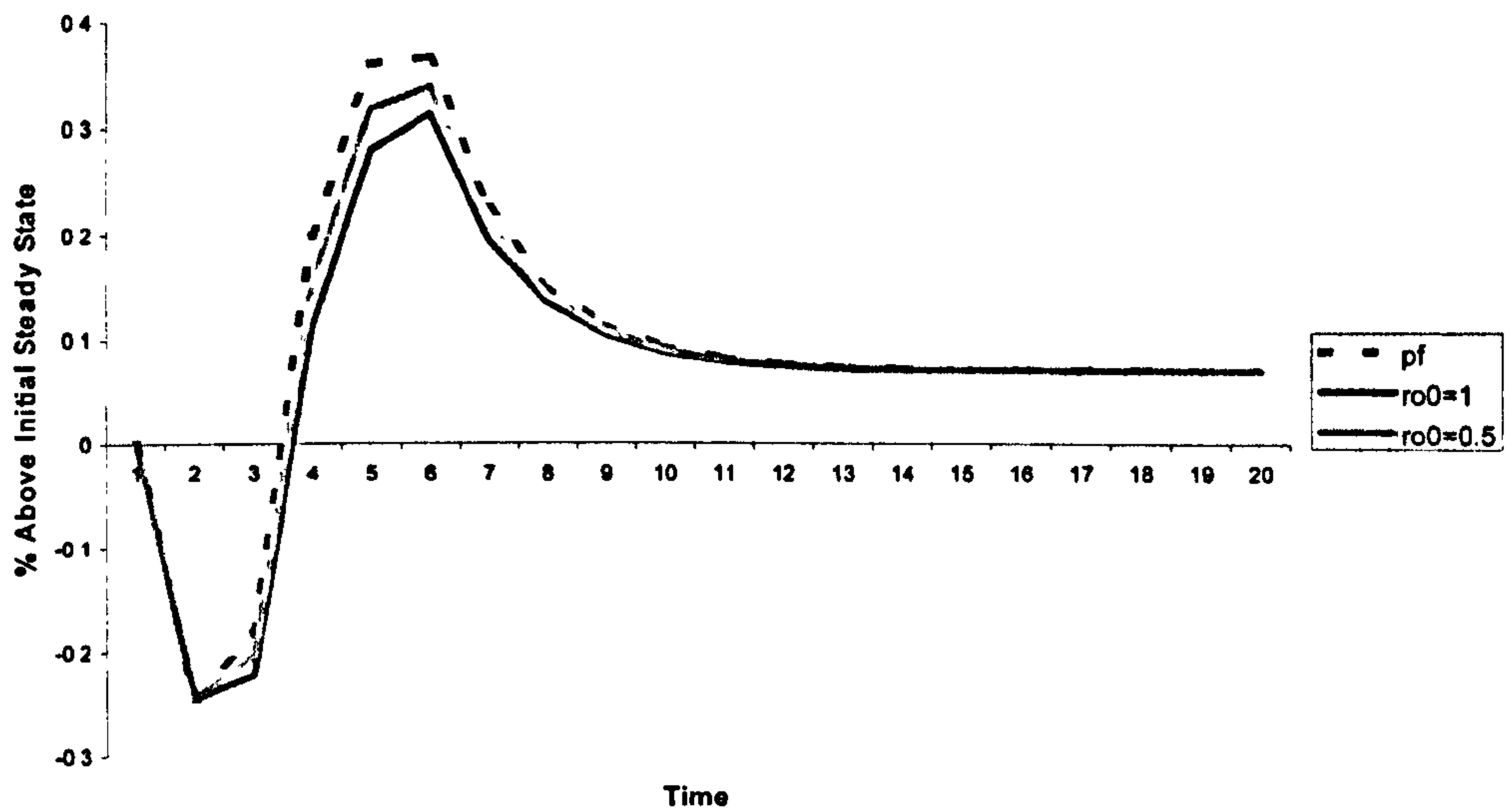


Figure 6-30: Output Effects of Gradual Disinflation under Convex Learning. Initial Annual Inflation Rate 3%.  $T=6$ . 3 Years to Perfect Foresight. R1.  $ro0=1$ .  $ro0=0.5$

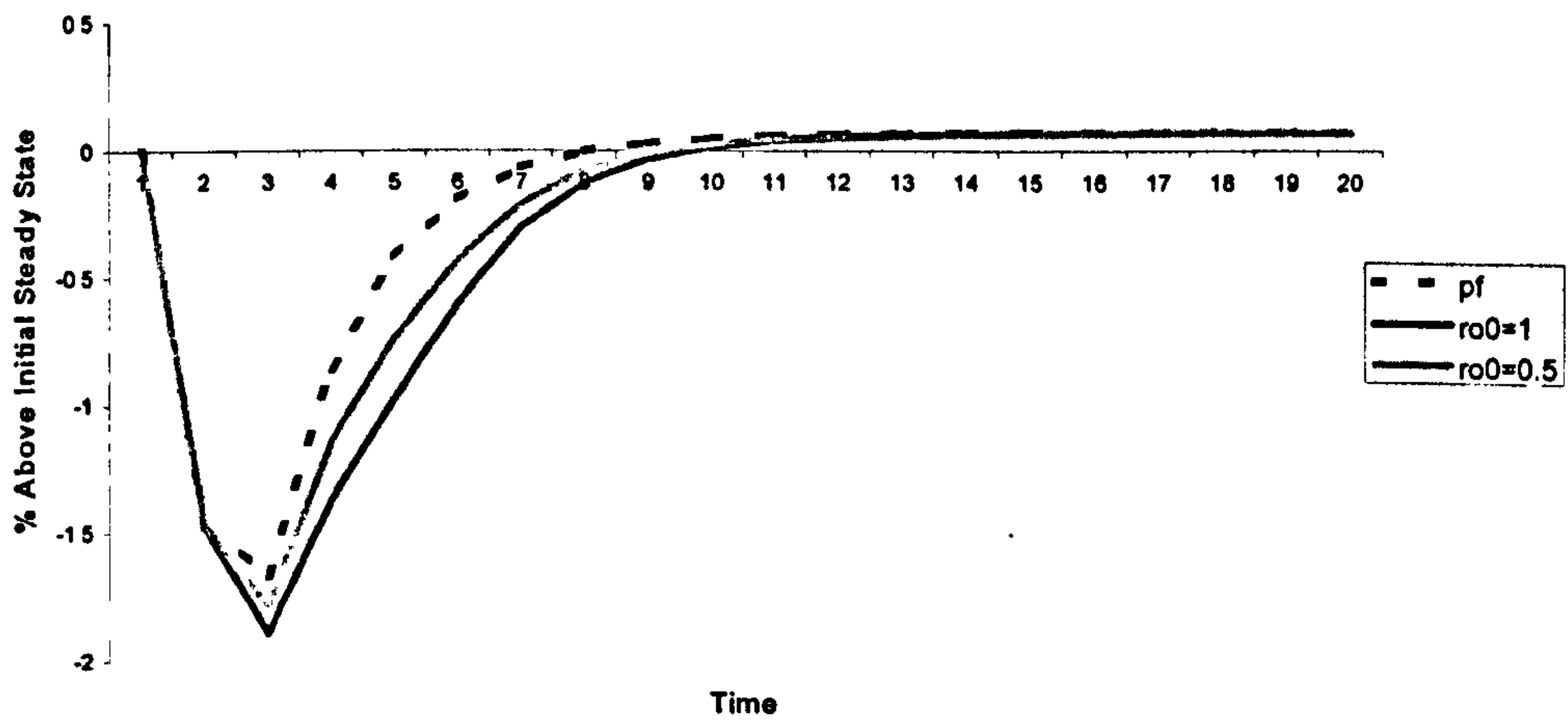


Figure 6-31: Output Effects of Immediate Disinflation under Concave Learning. Initial Annual Inflation Rate 3%. 3 Years to Perfect Foresight. R1.  $ro0=1$ .  $ro0=0.5$

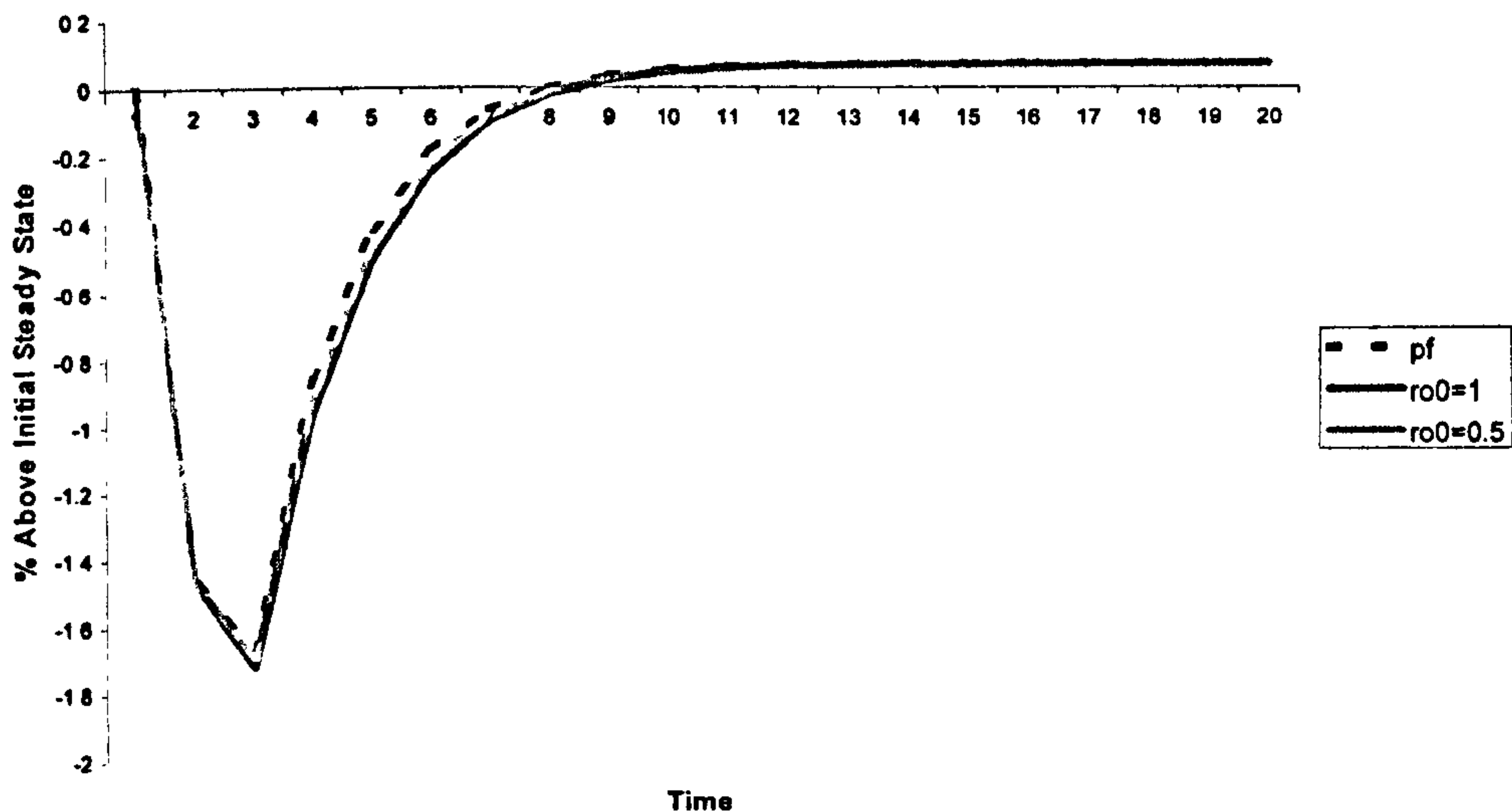


Figure 6-32: Output Effects of Immediate Disinflation under Convex Learning. Initial Annual Inflation Rate 3%. 3 Years to Perfect Foresight. R1.  $\rho_0=1$ .  $\rho_0=0.5$

Under convex learning, for the case of  $\rho_0 = 0.5$  the output is closer to that of perfect foresight ( $\rho_0 = 1$ ), but there is not a significant difference between these two cases, as seen in figure 6-30. However, the higher the level of credibility of the private sector at the beginning of the change in policy, the less painful the disinflation process will be in terms of output loss.

It is also clear that the type of learning can prevail over a low level of credibility. This shows that if the private sector learns fast at the beginning of the regime shift, even if there is absolute lack of credibility at the first moment, the loss in output is reduced and the cost of disinflation is smaller in comparison to the case in which there is a certain initial level of credibility at the moment of change in policy but the private sector is reluctant to believe the announced path of inflation (money supply). Therefore, even if there is an absolute lack of credibility at the moment of the regime shift, if the policymakers are feeding the private sector trust so as their policy becomes credible, the disinflation can be realized with smaller output loss.

The same conclusion is drawn regardless of the initial annual inflation rate, the length of the disinflation (figures 6-31 and 6-32) and the length of the period of time over which learning takes place. Under the second expectation updating, the output paths are literally almost identical.

## 6.4 Discussion and Conclusions

This section offers some concluding remarks on the behaviour of output under a disinflationary policy when there is temporary imperfect credibility, and on how the results obtained under imperfect foresight are different from those obtained under perfect credibility. This section also draws conclusions on how the component parts of the learning process (as defined in this work) are influencing these results.

As discussed in Chapter 2, the two main results standing out from the literature that have been further explored in this chapter are:

1. In the periods following a contraction in the money stock, real output is likely to fall below its long-run equilibrium level.
2. A gradual disinflation may actually result in output, after an initial decline, rising above its new steady-state level, and remaining high for some time.

From this study analysing the behaviour of output during a disinflationary process under imperfect credibility, it can be observed that:

1. The impact of imperfect credibility is to make disinflation more costly in terms of output losses in the period immediately following the contraction in monetary growth; the path of output to its new steady state differs from the path under perfect foresight. That cost seems likely to be more pronounced under concave updating as agents adjust initially only slowly to the announced new path for the money supply.
2. Nevertheless, the main result arising from this work is that here is demonstrated that the 'disinflationary boom' found by Ball (1994) and King and Wolman (1999) may or may not disappear in an environment with imperfect credibility. The conclusion is that their appearances depends on the speed of learning relative to the speed of disinflation.

However, in order to be able to provide the results of the impact of imperfect credibility on the behaviour of output in this chapter, the output effect of each of the factors determining the learning process has been analysed. Comparing *the two types of non-linear learning* under each of the expectation updating (4.3) and (4.4), the

way the private sector is learning at the beginning of the change in policy is essential. Disinflation under concave type of learning is more costly as agents adjust initially only slowly to the new announced path of the money supply. Moreover, under the convex path they avoid some of the more costly mistakes earlier in the disinflation in comparison to concave learning. Thus, regardless of the expectation updating, if the learning process is speeded up at the beginning of the disinflation process, then disinflation is achieved with minimal output loss.

Furthermore, the output effects of the type of learning are affected depending on *the expectations updating*. Under the second expectation updating these are reduced. The cost of disinflation encountered under concave learning under the second rule is diminished, since the initial expectational errors are much reduced. Under the second rule, the concave paths of output are extremely close to those of perfect foresight. In the light of these result, deflationary booms are more likely to occur under (4.4) and, as before, concave updating tends to make these booms disappear. Under either concave or convex learning, and especially under the former, the policymakers can improve the output gain if they manage to make the private sector think that the inflation rate that will occur is that of the last period rather than that of the beginning of the change in policy. If they can influence the private sector to believe that the inflation they will get at least equals that of the last period, then the output gain is improved. Any level of inflation the private sector thinks it is most likely to get other than at the moment of the change in policy is preferable with respect to the output gain. This conclusion can be seen as expanding on the third point of the conclusions of the previous section. This section argued that the expectation updating rule has a stronger influence on the behaviour of output than the type of learning involved.

Regarding *the length of the learning period over which perfect foresight is achieved*, this is directly related to the length and severity of the recession and the cost of disinflation. This is valid under either concave or convex type of learning, regardless of the expectation updating and is of a larger magnitude under concave learning. Also it can be observed that for learning periods greater than 5 years and small inflation rates output registers a cyclical behaviour. Although the (negative) output effects of the public being reluctant to believe the government's announcement at the beginning of the regime change, are minimized under the second expectation updating, these are

propagated in the future, due to an overlapping effect.

The level of trust the private sector has in the announcement of the monetary authority at the beginning of the change in policy is reflected in the disinflation process in terms of output loss, and the learning type can prevail over an initial low level of credibility. Even if there is absolute lack of credibility at the moment of the regime shift, if the policymakers are feeding the private's sector trust so their policy becomes credible, the disinflation can be realized with smaller output loss. Also the way the expectations can be influenced can make a change. Obviously, the shorter the learning process, the better.

Results standing out from the investigation are:

1. First, imperfect credibility and price stickiness are jointly neither necessary nor sufficient for monetary contraction to cause lengthy recessions, at least for the relatively modest initial levels of inflation under scrutiny here. In this sense these results conflict with Ball (1995).
2. Second, in this model imperfect credibility need not be an overriding concern for policymakers initiating a period of disinflation; the disinflationary booms still appear.

Central to both of these results is the size of the expectational errors early on in the regime shift relative to the degree of price stickiness. For a given level of imperfect credibility, longer nominal contract length will generally imply sharper recessions following a monetary contraction. Under convex updating these initial errors are relatively small while under concave updating they are considerably larger.

Future research might investigate two issues. First, it would be interesting to know how large does the initial fall in output have to be before it is optimal for the policymakers to renege on the announcement to attain price stability. This issue is accommodated by focusing on situations in which the initial rate of inflation was relatively modest. For countries with very high initial inflation rates, however, renegeing might well be optimal, as figure 6-8 strongly suggests. A second issue concerns the interaction between monetary and fiscal policy. This issue is side-stepped altogether by ignoring distortionary taxation of real activity. As initial inflation rises, the public sector budget constraint is likely to play an increasing role in the welfare calculus.

# Chapter 7

## Optimal Speed of Disinflation under Imperfect Credibility

### 7.1 Introduction

At the centre of this chapter lies the optimal speed of disinflation, a topic which, as presented in the literature review, so far has been discussed only by Ireland (1997). Nevertheless, in his model Ireland considers an economy under the case of perfect foresight. However, no work has been done so far to determine the optimal speed of disinflation under the case of imperfect credibility.

This research quantifies the impact of the temporary lack of credibility on the optimal speed of disinflation and suggests that big inflations should be stopped quickly, in cases of both perfect and imperfect credibility, but for the case of imperfect credibility, the speed of disinflation of small inflations is decreased compared to the case of perfect foresight. It also suggests that once inflation has risen substantially, imperfect credibility makes sizeable output losses in the transition to price stability highly likely, even when the speed of disinflation is 'optimal'.

The outline of this chapter is as follows: Section 7.2 describes the way the optimal speed of disinflation has been calculated. Section 7.3 analyses the costs imposed by imperfect credibility on the optimal speed of disinflation and finally, in Section 7.4 we enquire whether the speed of the Volcker disinflation was excessive or not. Section 7.5 concludes.



## 7.2 Methodology

In this section the way the optimal speed of disinflation has been calculated for both cases of perfect foresight (Ireland (1997)) and imperfect credibility is presented. The methodology is similar as the only difference consists in the conditions under which output and the labour supplied entering the utility function have been determined: perfect foresight or imperfect credibility.

First the way the optimal speed of disinflation has been calculated by Ireland (1997) for the case of perfect foresight is presented. As seen in Chapter 3, in his model, Ireland employed the Ramsey utility criterion (3.1). In order to determine the optimal speed of disinflation, for every initial annual inflation rate between 1% and 250 % a series of Ramsey utilities is calculated while the disinflation is made over a period of time, ranging from  $T = 1$  (immediate disinflation) to  $T = 31$  (more gradual), where  $T$  is the length of the period of disinflation measured in half yearly intervals. This gives the length and the severity of the monetary contraction, as given by (3.19). Thus, for a given initial annual inflation rate, the utility is calculated for each length of the disinflation period. From this range of lengths of periods of disinflation, the one for which the Ramsey utility attains the maximum value is selected as optimal.

As can be seen from figure 7-1, Ireland (1997) finds that for a small initial annual inflation rate, disinflation is best done gradually: the optimal length of the period of disinflation is  $T = 31$ , corresponding to no less than 15 and half years. For higher inflations the length of the optimal period of time is reduced.

The intuition behind this is clear when observing the behaviour of output under various lengths of the disinflation period. The inverse relation between the output loss (measured by the area delimited by output and found below the initial steady state) and the length of the period of disinflation when disinflating small inflations changes for the case of big inflations.

### *For small initial annual rates of inflation*

If  $T = 1$ , which corresponds to the case in which disinflation is realized in a immediate manner, the output loss is bigger than the area described by the evolution of output when the disinflation is done more gradually. From figure 6-4 it can be observed that for a 3% initial annual inflation rate, the more gradual the disinflation, the smaller

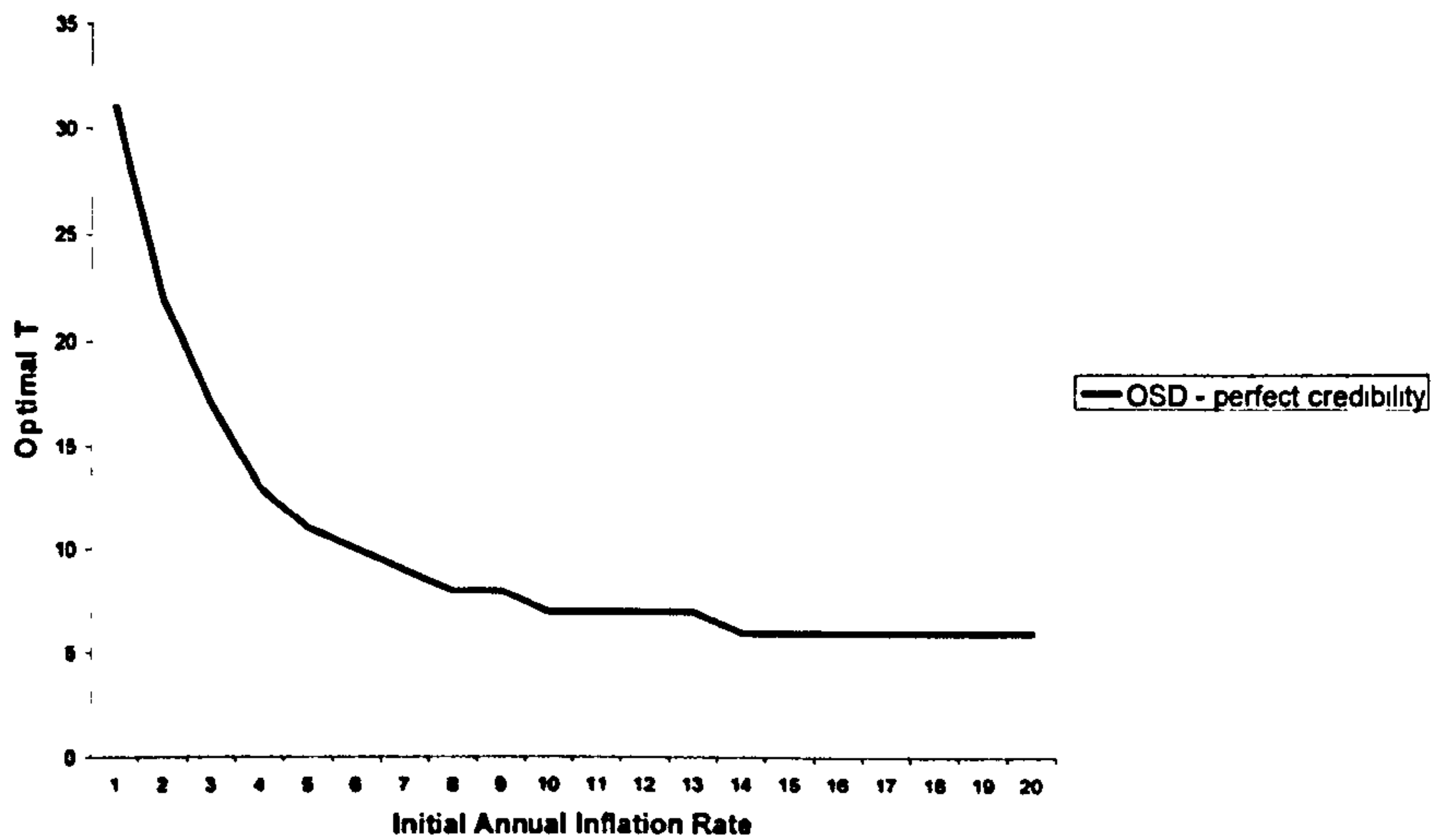


Figure 7-1: Optimal Speed of Disinflation under Perfect Credibility

the output loss.

*For higher initial annual rates of inflation*

If for the previous instance there was an inverse relationship between the output loss and the length of the disinflation period, for the case of higher inflations, this relationship is changed. As seen from figure 7-2, there is a direct relation now between the length of the period of disinflation and the output loss. Of course, the utility calculation considers not only the output, but also the cost of the labour supplied. This does not affect the results, as the labour is proportionally altered with the price, related linearly to output. Therefore, it can be concluded that it is optimal to end small inflations gradually and big inflations more quickly.

For the calculation of the optimal speed of disinflation under the case of imperfect credibility the same method is employed as in the case of perfect foresight, only that now, the two variables entering the Ramsey Utility function (the output and the labour supplied) are altered to include the effects of the learning process during a temporary period of imperfect credibility, as discussed in Chapter 4.

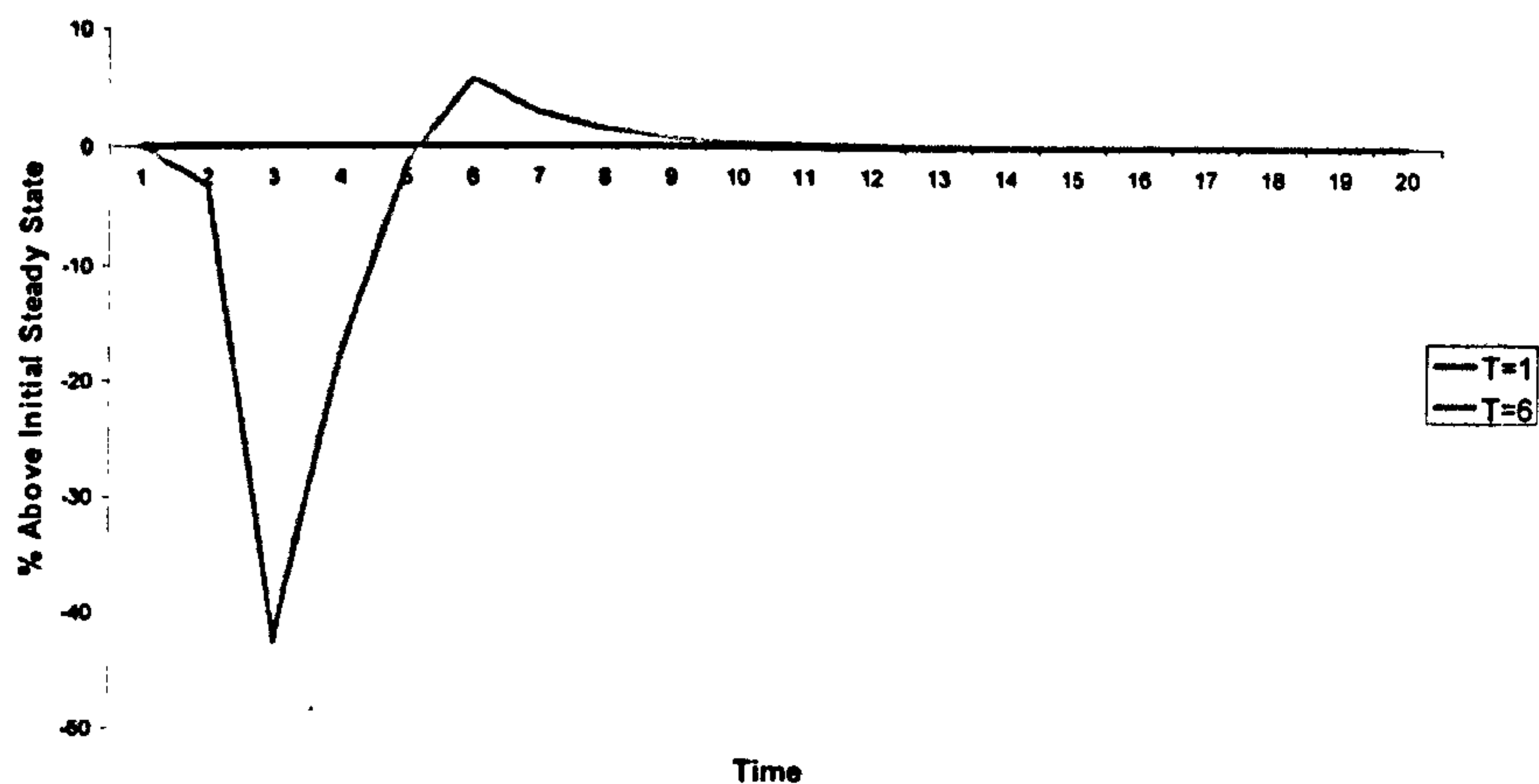


Figure 7-2: Output Effects of Gradual Disinflation under Perfect Foresight. Initial Annual Inflation Rate 200%.  $T=1$ .  $T=6$

### 7.3 The Effect of Imperfect Credibility on the Optimal Speed of Disinflation

Ireland (1997) found the relationship between the initial annual inflation rate and the length of disinflation when disinflation is realized with minimized loss of welfare under perfect foresight. In this section there is an attempt to determine how this relationship is going to be altered under the case of imperfect credibility and to identify which of the factors determining the learning process influence this relationship and how. Given the extra cost imposed by imperfect credibility, the question that arises is what is the extent of the quantitative impact on the optimal speed of disinflation.

While undertaking this exercise, the evolution of the optimal speed of disinflation under the two types of updating expectations (4.3) and (4.4), have been looked at. Each is observed under the two types of learning discussed: concave and convex. If in updating their expectations agents believe that the next period's inflations will equal the previous period's, the optimal speed of disinflation is the same under imperfect credibility as in perfect foresight. However, the costs imposed by imperfect credibility affect the optimal speed of disinflation if in their expectation updating agents perceive the authorities as reverting to the previous steady state inflation. Moreover, the size of the expectational errors early on in the regime shift relative to the degree of stickiness

also matters, as the optimal speed of disinflation under convex learning is closer to the case of perfect foresight than under concave learning.

The effect the length of the learning period and the impact the initial level of lack of credibility may have on the optimal speed of disinflation have also been analysed. The conclusion is that both affect the optimal speed of disinflation but only for small initial annual inflation rates like 2% and 3%. If the learning process takes longer than one year (from 3 to 4 years), the optimal speed of disinflation is decreased only when disinflating a 2% annual inflation rate, under the assumption that the learning process of the private sector starts from absolute lack of credibility. If there is a lower level of the lack of credibility (half measure) the speed of disinflation is increased compared to the case in which there is absolute lack of credibility for the annual inflation rates of 2% -3%, for the case in which learning takes 3 years.

### **7.3.1 The Effect of the Expectation Updating on the Optimal Speed of Disinflation**

By calculating the optimal speed of disinflation under both expectation updating (4.3) and (4.4), it occurs that this is not affected by the costs imposed by imperfect credibility if the agents expect the disinflation policy to stall, in other words if they expect an inflation rate equal to that of the previous period. Indeed, under the second rule, the expected money supply is closer to that announced by the monetary authority and as expected, the path of output is much closer to that achieved under the case of perfect credibility. Therefore, the costs imposed by imperfect credibility are insignificant on this occasion and as a result, the optimal speed of disinflation is identical to that achieved under the case of perfect foresight.

However, this is no longer the case if the expectation updating rule employed captures the case in which agents perceive the inflation rate as being that of the previous steady state. Under the first expectation updating rule, the costs imposed by imperfect credibility affect the optimal speed of disinflation and this happens for both types of learning: concave and convex. Therefore, in the rest of this chapter, will only be presented the analysis of the costs imposed by imperfect credibility on the optimal speed of disinflation calculated under the first expectation updating rule (4.4).

### 7.3.2 The Effect of the Learning Type on the Optimal Speed of Disinflation

When analysing the influence of the learning type on the optimal speed of disinflation, the optimal speed of disinflation is calculated assuming that the private sector is learning from absolute lack of credibility over a period of 3 years to arrive at perfect foresight. The size of the expectational errors early on in the regime shift relative to the degree of stickiness matters, as the optimal speed of disinflation under convex learning is closer to the case of perfect foresight in comparison with concave learning. As the initial inflation rate rises, the contraction in output in the early periods of the disinflation is more pronounced, increasingly offsetting the utility gain from the subsequent boom and therefore the optimal speed of disinflation rises. Under imperfect credibility, the initial contraction in output is more severe for any initial inflation rate than in the case of perfect foresight. Furthermore, the utility gain from the disinflationary boom is absent. It appears that up until an initial rate of around 12% a more gradual period of disinflation is optimal. The optimal speed of disinflation is the same as under perfect foresight for initial inflation rates greater than 12%.

Therefore, under perfect foresight, gradual disinflations are primarily about reaping the utility from output gains following an initial contraction in activity, while under imperfect credibility, they are primarily aimed at avoiding over-sharp contractions in activity in the early period of the disinflation. In this sense, the optimal speed of disinflation is crucially different between perfect foresight and imperfect credibility. Consequently, the optimal speed of disinflation under both types of non-linear learning will be further presented.

For the case of **concave learning**, figure 7-3 reveals that a good 'rule-of-thumb' is that disinflation from initial rates between 2% -11% should take an extra year, compared with the case of perfect foresight. For inflation rates above 12% and less than or equal to 1%, the optimal speed of disinflation is indistinguishable from that of perfect foresight. The key reason that gradual disinflations are attractive is that, with some price stickiness and perfect foresight, they often imply prolonged periods of above trend output and consumption.

On the other hand, for the case of **convex learning**, as presented in figure 7-4, the optimal speed of disinflation is closer to the case of perfect foresight in comparison

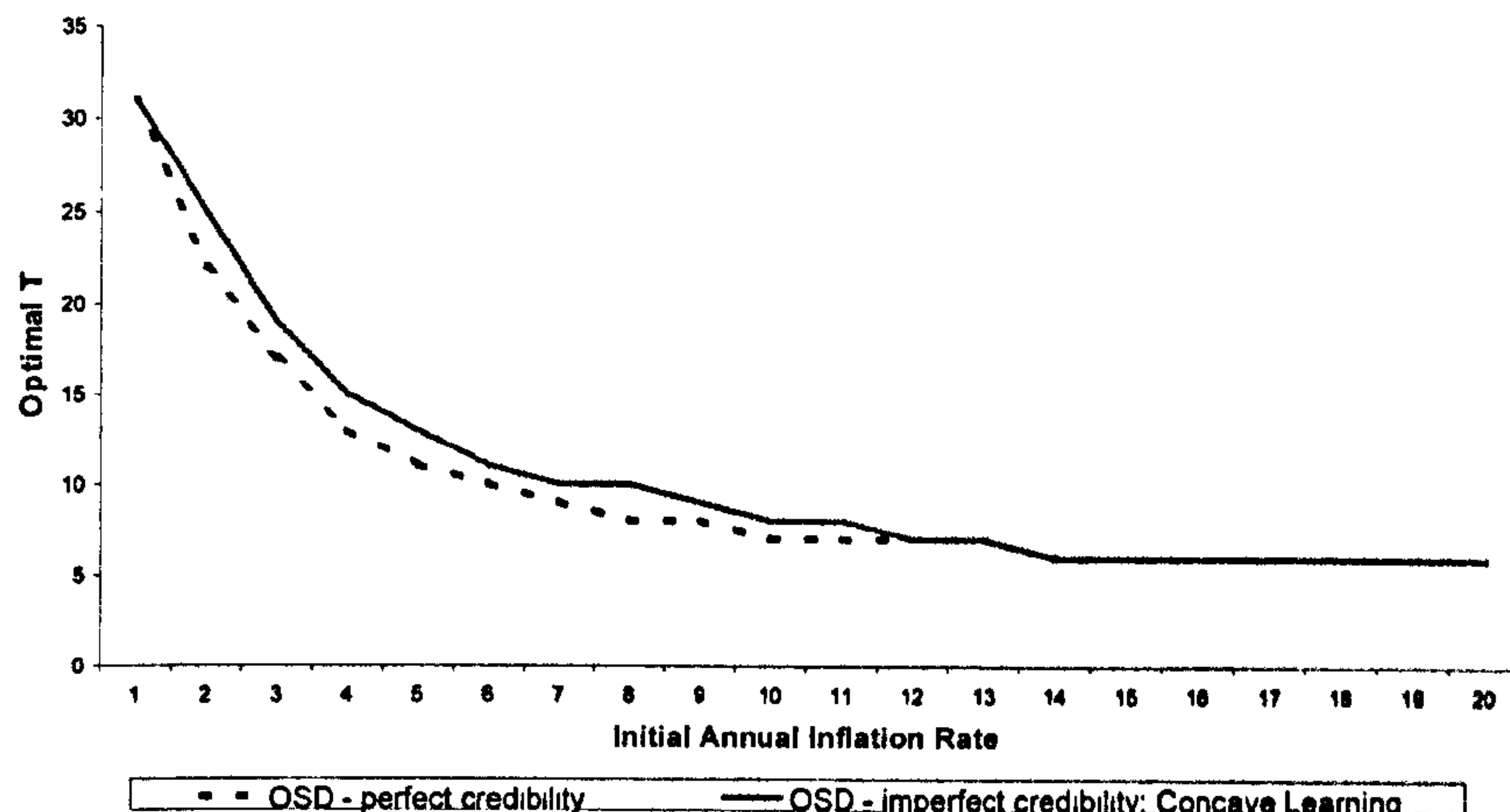


Figure 7-3: The Effect of Imperfect Credibility / Concave Learning on the Optimal Speed of Disinflation

with concave learning. This is due to the size of the expectational errors early on the regime shift relative to the degree of stickiness. Disinflation should be more gradual for inflation rates between 3% and 11% as in this case the optimal speed of disinflation takes one year longer. For annual inflation rates of 1% -2% and thereafter 12%, the optimal speed of disinflation is identical to that of the case of perfect foresight. For the annual inflation rates of 3%, 6%-11% and 4%-5%, the speed of disinflation decreases by half a year and one year respectively.

The difference between concave and convex learning is seen only for the annual inflation rates of 2% and 3%, as under concave learning the optimal speed of disinflation is further decreased in comparison with convex learning. The way the agents are learning at the beginning of the change in policy affects the optimal speed of disinflation for small annual inflation rates of 2%-3%. For annual inflation rates of 1% and 4% onwards the costs imposed by imperfect credibility have the same effect on the optimal speed of disinflation regardless of the type of learning.

### 7.3.3 The Effect of the Length of the Learning Period on the Optimal Speed of Disinflation

To analyse the impact of the length of time over which learning takes place on the optimal speed of disinflation, it is assumed that the private sector starts the learning

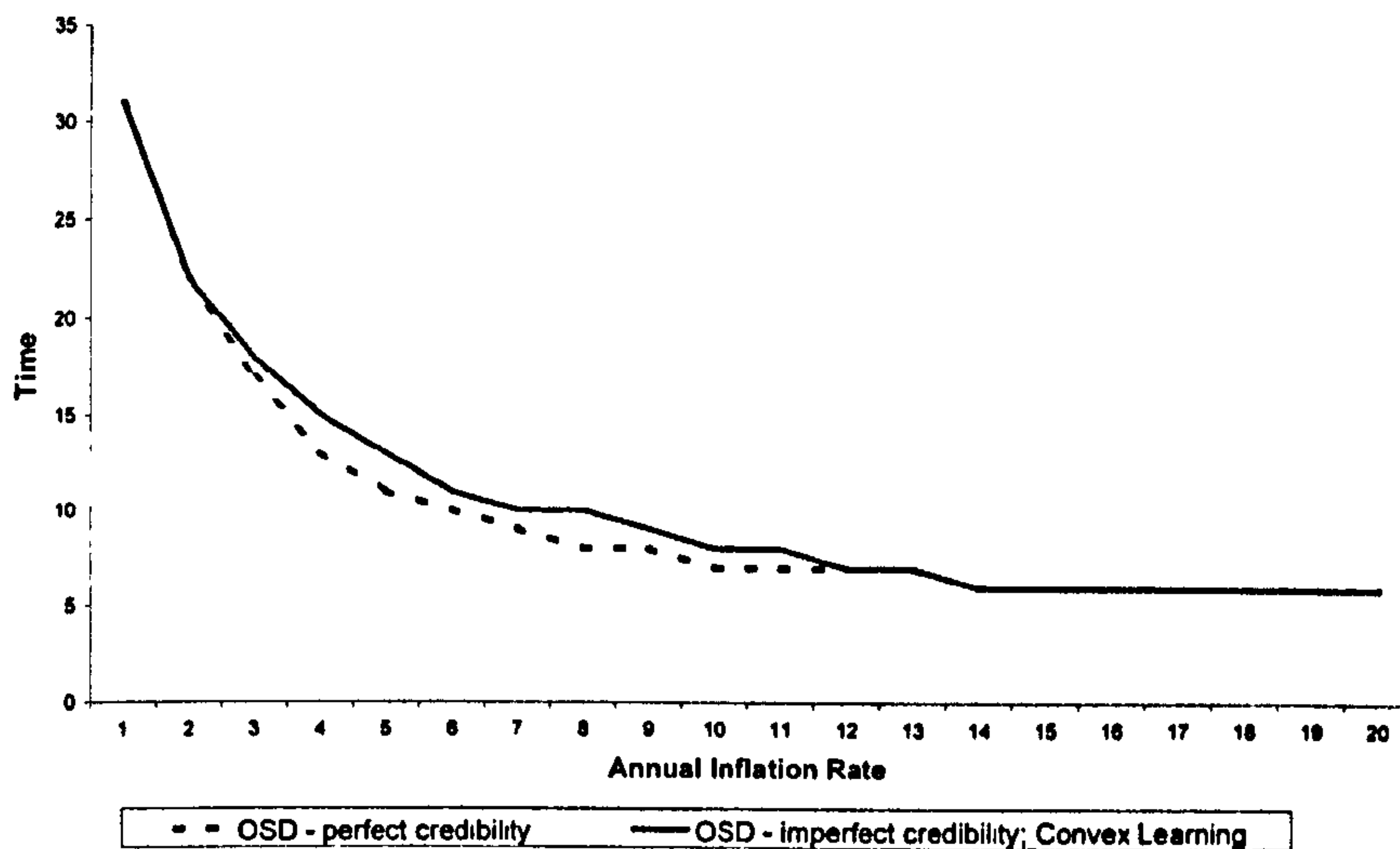


Figure 7-4: The Effect of Imperfect Credibility / Convex Learning on the Optimal Speed of Disinflation

process from absolute lack of credibility and achieves perfect credibility in 3 and 4 years. It is also assumed that there is concave learning. As expected, if the length of the learning period over which perfect foresight is achieved is longer, the optimal speed of disinflation will be slower, but this happens only for the case in which the initial annual inflation rate is 2%. In this case it takes half a year longer for the disinflation to take place 'optimally', as seen in figure 7-5. For all the other annual inflation rates, the optimal speed of disinflation is identical to the case in which learning takes place over 3 years. In this case, an increase of only one year in the time over which the private sector learns does not affect the speed of the optimal disinflation except for the 2% annual inflation rate. However, a longer learning period may have a stronger effect.

### 7.3.4 The Effect of the Initial Level of Lack of Credibility on the Optimal Speed of Disinflation

To analyse the effect of the initial level of lack of credibility on the optimal speed of disinflation it is assumed that the private sector starts the learning process first, from absolute lack of credibility and then from a measure of half level of lack of credibility. It is assumed that they attain perfect credibility over a period of 3 years and that there is concave learning.

It is expected that the optimal speed of disinflation will be faster if the initial level

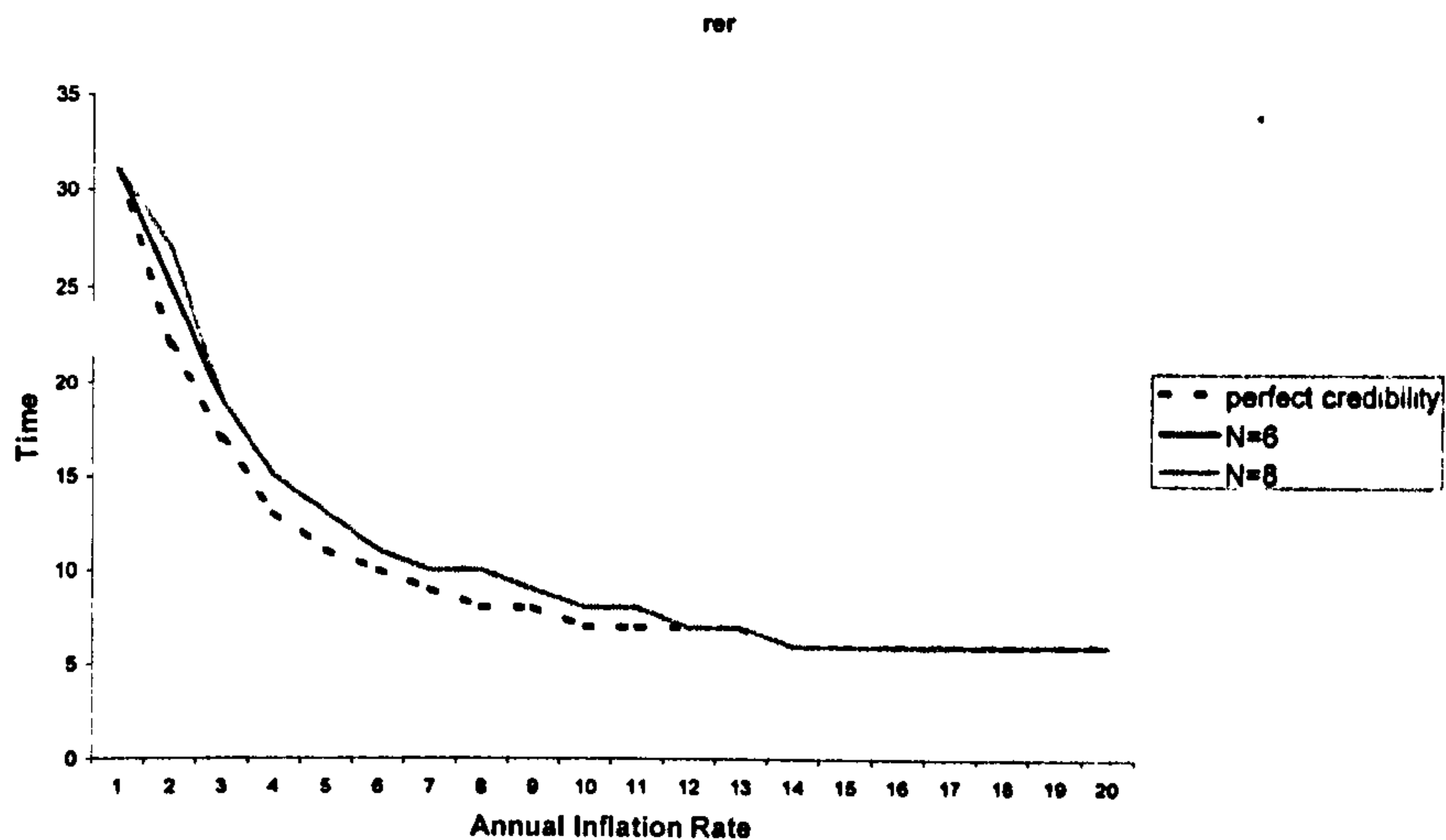


Figure 7-5: The Effect of the Length of Learning Period on the Optimal Speed of Disinflation. Concave Learning.

of lack of credibility is smaller compared to the case in which there is an absolute lack of credibility. For the framework chosen to conduct the analysis, a lower initial level of the lack of credibility increases the speed of the disinflation for the annual inflation rates of 2% and 3% with one and half a year respectively, as seen in figure 7-6. For all the other annual inflation rates, the optimal speed of disinflation is identical to the case in which there is initial absolute lack of credibility. Overall, calculated for the case in which the initial level of the lack of credibility is half, the optimal speed of disinflation is closer to that achieved under perfect foresight. For the framework in which this analysis is conducted, the optimal speed of disinflation is not much affected by the decrease in the initial level of the lack of credibility except for the small annual inflations of 2%-3%.

This can be explained by the fact that this area of small inflations is more sensitive to changes and also more responsive to changes in the initial level of the lack of credibility and the length of the period over which learning takes place. The scale of the graduality of the money supply contraction is not much affected by the length over which disinflation takes place, and so a change in the learning period over which perfect foresight is achieved or the initial level of the lack of credibility affects the behaviour of output during disinflation thus making a change to the optimal speed of disinflation. It can be observed that small inflations are more demanding as they are more difficult



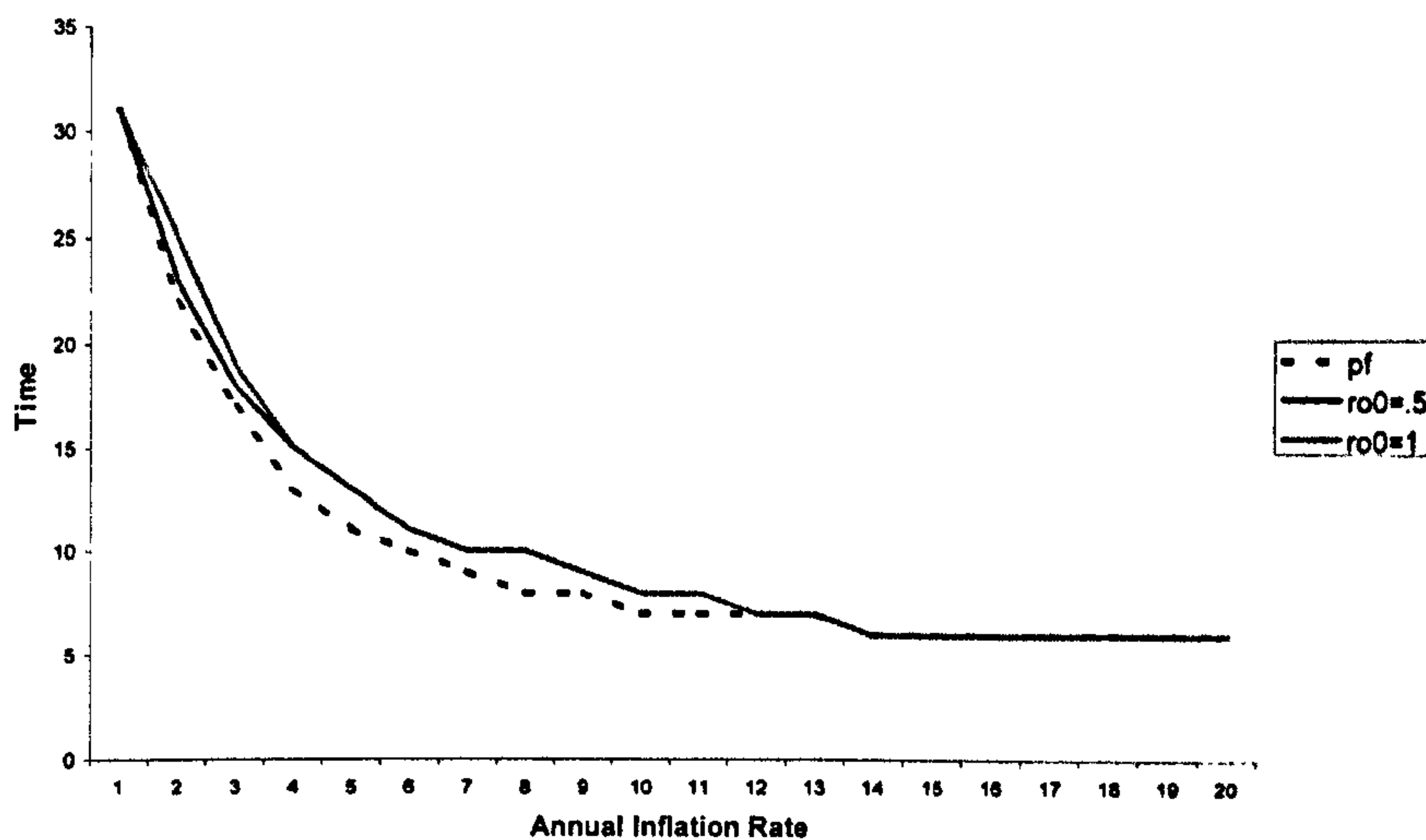


Figure 7-6: The Effect of the Initial Level of Lack of Credibility on the Optimal Speed of Disinflation

to bring to a halt in the 'optimal' period of time corresponding to perfect foresight. As seen, they can easily be affected by any of the factors determining the learning process, making the new optimal period of disinflation under imperfect credibility longer.

## 7.4 The Volcker Disinflation and the Optimal Speed of Disinflation

In this section an attempt is made to come up with what might be an empirically plausible version of the learning rules adopted in Chapter 4. This is done by using the experience of the US in the early 1980s.

Between late 1979 and 1985 inflation in the US fell from 10% to 4%, and has subsequently fallen further. As the measures of the output gap from this period indicate, this reduction in the inflation rate was not inexpensive. This work raises the question whether the Volcker disinflation may have been too rapid. To answer this, it has been assumed that the initial steady state inflation was 10% (around the highest level that actual US inflation reached before coming down), and that Volcker intended to reduce inflation to 2% (close to the average inflation rate since 1984).

The currency component of M1 is used as the path for the money stock during the disinflation, from 1979 until 1984. Given this path and assuming perfect foresight, the

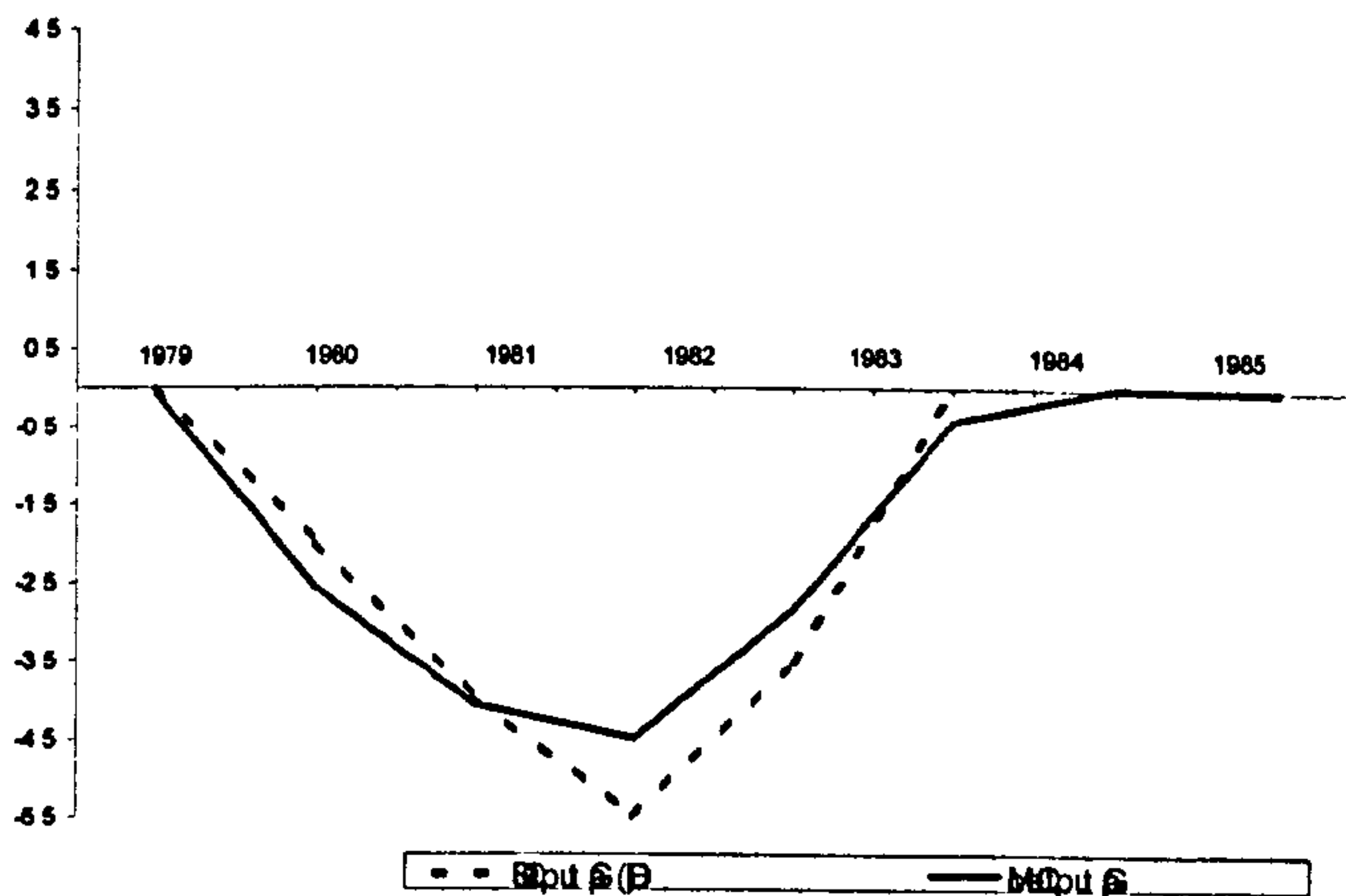


Figure 7-7: Volker Disinflation

flexible price level of output of the model. A learning process is backed out so that the output gap in the model tracks the actual US output gap. Given the implied learning process, the optimal speed of disinflation is subsequently calculated.

In order to generate such a sharp contraction in output like that of the US output gap measured by the OECD, a rather extreme form of concave learning had to be adopted. It was assumed that for almost the whole course of the disinflation (5 years), agents do not believe the 'announced' money supply growth rates and then in the course of year six, they believe completely the announced path. Even under this extreme form of concave learning, the sharpness of the contraction indicated in the OECD data cannot quite be generated; although both measures of the output gap bottom out in 1982, the model output gap at the trough is 1% less than the OECD estimate.

The optimal speed of disinflation from an initial rate of inflation of 10% to 2% under perfect foresight is three and half years. In contrast, the optimal speed under the learning process implied above is seven years. The calculations suggest that even had the disinflation taken longer than it actually did the output losses would have been only marginally lower.

Clearly one needs to be cautious in an exercise such as this, as real world data are effectively squeezed into a highly simplified framework, but these results are indicative. It appears that once inflation has risen substantially, imperfect credibility makes

sizeable output losses in the transition to price stability highly likely, even when the speed of disinflation is 'optimal'. In a related analysis Erceg and Levin (2003) come to a similar conclusion. They examine the Volcker disinflation under the assumption that agents use optimal filtering to disentangle persistent changes in the inflation target from temporary shocks to the monetary policy rule. Unlike the current set up, in their model the steady state is unchanged throughout this period. However, leaving aside this important difference (it may be important for the sorts of reasons discussed in Albanesi, Chari and Christiano (2003)), they also find that sizeable output losses may be the price of imperfect credibility.

## 7.5 Conclusions

This research suggests that big inflations should be stopped quickly, under both perfect and imperfect credibility. However, in the case of imperfect credibility, the speed of disinflation of small inflations is decreased compared to the case of perfect foresight. It turns out that a more gradual period of disinflation is optimal up until an initial rate of around 12%. For initial inflation rates greater than 12% the optimal speed of disinflation is the same as under perfect foresight.

The optimal speed of disinflation is not really affected by the costs imposed by imperfect credibility if the agents expect the disinflation policy to stall, while this is no longer valid if they expect the inflation rate to be that of the previous steady state. In this later case, the optimal speed of disinflation is changed for both concave and convex types of learning.

However, the optimal speed of disinflation under concave learning differs from that under convex learning only for the annual inflation rates of 2% and 3%. The difference is that under concave learning the optimal speed of disinflation is further decreased in comparison with convex learning. For annual inflation rates of 1% and 4% onwards, regardless of the type of learning, the costs imposed by imperfect credibility have the same effect on the optimal speed of disinflation.

Nevertheless, the type of learning is not the only one that decreases the optimal speed of disinflation. This is decreased for the small annual inflations of 2%-3% also by an (one year) increase in the length of the learning process and by the initial level of lack of credibility, while it stays the same for any other initial rates of inflations.

This research also suggests that once inflation has risen substantially, even when the speed of disinflation is 'optimal', imperfect credibility makes sizeable output losses in the transition to price stability highly likely.

# Chapter 8

## Conclusions

The main research contribution of this thesis is the investigation of the impact of a temporary lack of credibility in a transition to price stability. The analysis takes place in a New-Keynesian model with sticky prices. The supply-side of the economy is characterized by monopolistically competitive firms and there is rigidity in the setting of prices. The effects of a disinflationary monetary policy are studied when policy makers are committed to price stability. Moreover, I propose a new and more flexible way of modelling the evolution of agent's priors.

My thesis addresses the impact of imperfect credibility on:

- (1) The effects of a period of disinflation on temporary output losses and;
- (2) The optimal speed of disinflation.

Regarding (1):

(a) Under imperfect credibility, the path of output to its new steady state differs from the path under perfect foresight. The impact of imperfect credibility is to make disinflation more costly. In the periods following a contraction in the money stock, real output is likely to fall below its long-run equilibrium level. The fall is even bigger under the impact of imperfect credibility, making the disinflation more costly in terms of output losses. This cost seems likely to be more pronounced under concave updating as agents adjust initially only slowly to the announced new path for the money supply.

(b) The 'disinflationary boom' found by Ball (1994) and King and Wolman (1999) may or may not disappear in an environment with imperfect credibility. The 'disinflationary boom' depends on the speed of learning relative to the speed of disinflation.

In the light of these two central results, I conclude: (i) Imperfect credibility and price stickiness are jointly neither necessary nor sufficient for monetary contraction to cause lengthy recessions, at least for the relatively modest initial levels of inflation generally under scrutiny here. In this sense these results conflict with Ball (1994). (ii) In this model imperfect credibility need not be an overriding concern for policymakers initiating a period of disinflation; the disinflationary booms still appear.

Regarding (2):

(a) In both cases of perfect and imperfect credibility big inflations should be stopped rapidly.

(b) In the case of imperfect credibility, for small inflations, the speed of disinflation decreases in comparison to the case of perfect foresight.

(c) Examining whether the speed of the Volcker disinflation was excessive or not, this research also suggests that once inflation has risen substantially, even when the speed of disinflation is 'optimal', imperfect credibility makes sizeable output losses in the transition to price stability highly likely.

Future research based on my thesis may address the following three issues:

- First, how large does the initial fall in output have to be before it is optimal for the policymakers to renege on the announcement to attain price stability. We accommodate this issue by focusing on situations where the initial rate of inflation was relatively modest. For countries with very high initial inflation rates, however, reneging might well be optimal, as figure 6-8 strongly suggests.
- A second issue concerns the interaction between monetary and fiscal policy. This issue is side-stepped altogether in this research by ignoring distortionary taxation of real activity. Again, so long as initial inflation, and hence seigniorage, is relatively modest the fiscal implications of disinflation are probably limited. As initial inflation rises, the public sector budget constraint is likely to play an increasing role in the welfare calculus.

- A final issue concerns how quickly agents actually take to learn about regime shifts. In this paper was picked what thought to be sensible scenarios. In truth, relatively little about such issues is known, although in Chapter 7, Section 4 a tentative first step in this direction is made. Given its importance to this result, this is an important topic for future research.

# Bibliography

- [1] Albanesi, Stefania, V. V. Chari and Lawrence J. Christiano (2003), "Expectations Traps and Monetary Policy" *The Review of Economic Studies*, Vol. 70 (4) No. 245, pp. 715-741.
- [2] Almeida, Alvaro and Charles A. E. Goodhart (1998), "Does the Adoption of Inflation Targets Affect Central Bank Behaviour?", *Banca Nazionale del Lavoro Quarterly Review. Special Issue*, March, Rome.
- [3] Almeida, Heitor and Marco Bonomo (2002), "Optimal State-dependent Rules, Credibility, and Inflation Inertia", *Journal of Monetary Economics*, Vol. 49, Issue 7, pp. 1317-1336.
- [4] Andolfatto, David and Paul Gomme (1999), "Monetary Policy Regimes and Beliefs", Federal Reserve Bank of Cleveland Working Paper, No. 9905.
- [5] Antulio, Bomfin and Glen Rudebusch (2000), "Opportunistic and Deliberate Disinflation under Imperfect Credibility", *Journal of Money, Credit, and Banking*, Vol. 32, No. 4 (November, Part 1), pp. 707-721.
- [6] Ball, Laurence, N. Gregory Mankiw and David Romer (1988), "The New Keynesian Economics and the Output-Inflation Trade-Off", *Brookings Papers on Economic Activity*, Vol. 1, pp. 1-65.
- [7] Ball, Laurence (1994), "Credible Disinflation with Staggered Price-Settings", *The American Economic Review*, Vol. 84, No. 1, pp. 282-289.
- [8] Ball, Laurence and N. Gregory Mankiw (1994), "Asymmetric Price Adjustment and Economic Fluctuations", *Economic Journal*, Vol. 104, March, pp. 247-261.
- [9] Ball, Laurence (1995), "Disinflation with Imperfect Credibility", *Journal of Monetary Economics*, Vol. 35, Issue 1, pp. 5-23.



- [10] Ball, Laurence (1997), "Efficient Rules for Monetary Policy", *NBER Working Paper*, No. 5952.
- [11] Ball, Laurence (1998), "Policy Rules for Open Economies", Reserve Bank of Australia Research Discussion Paper No. 9806.
- [12] Ball, Laurence (1999a), "Policy Rules for Open Economies" in *Monetary Policy Rules*, edited by John Taylor, University of Chicago Press for NBER, Chicago pp. 127-144.
- [13] Ball, Laurence (1999b), "Efficient Rules for Monetary Policy", *International Finance*, Vol. 2, No. 1, pp. 63-83.
- [14] Ball, Laurence (2000), "Policy Rules and External Shocks", *NBER Working Paper*, No. 7019.
- [15] Batini, Nicoletta and Andrew, Haldane (1999), "Forward-looking Rules for Monetary Policy" in: *Monetary Policy Rules*, edited by John Taylor, University of Chicago Press for NBER, Chicago, pp. 157-192.
- [16] Benabou, Roland and Jerzy D. Konieczny (1994), "On Inflation and Output with Costly Price Changes: A Simple Unifying Result", *American Economic Review*, Vol. 84, No. 1, March, pp. 290-297.
- [17] Bernanke, Ben S., and Frederic S. Mishkin (1997), "Inflation Targeting: A New Framework for Monetary Policy?", *Journal of Economic Perspectives*, Vol. 11, No. 3, pp. 97-116.
- [18] Bernanke, Ben S., Thomas Laubach, Frederic S. Mishkin, and Adam S. Posen (1999), *Inflation Targeting: Lessons from the International Experience*, Princeton, New Jersey.
- [19] Barro, Robert J. and David B. Gordon (1983), "Rules, Discretion, and Reputation in a Model of Monetary Policy", *Journal of Monetary Economics*, Vol. 12, No.1, July, pp. 101-121.
- [20] Blinder, Alan S. (1999), "Central-Bank Credibility: Why Do We Care? How Do We Build It?", *NBER Working Paper*, No. 7161.

- [21] Blinder, Alan S. (2000), "Central-Bank Credibility: Why Do We Care? How Do We Build It?", *American Economic Review*, Vol. 90, No. 5, December, pp. 1421-1431.
- [22] Bonomo, Marco and Carlos Carvalho (2004), "Endogenous Time-Dependent Rules and Inflation Inertia", *Journal of Money, Credit, and Banking*, Vol. 36, No. 6, pp. 1015-1041.
- [23] Bryant, Ralph C., Peter Hooper and Catherine L. Mann (1993), *Evaluating Policy Regimes: New Research in Empirical Macroeconomics.*, Washington, D.C, Brookings Institution.
- [24] Christoffersen, Peter F. and Robert F. Wescott (1999), "Is Poland ready for Inflation Targeting?", *IMF Working Paper*, No. 41.
- [25] Clark, Peter B, Douglas Laxton and David Rose (1995), "Capacity Constraints, Inflation and the Transmission Mechanism: Forward-Looking Versus Myopic Policy Rules", *IMF Working Paper*, No. 75.
- [26] Corbo, Vittorio and Klaus Schmidt-Hebbel (2000), "Inflation Targeting in Latin America", Paper presented at the *Latin American Conference on Financial and Fiscal Policies*, Stanford University, November.
- [27] Cukierman, Alex (1995), "Towards a Systematic Comparison Between Inflation Targets and Monetary Targets", in *Inflation Targets*, edited by Leiderman L and Lars E. O. Svensson, CEPR.
- [28] Corce, Enzo and Mohsin S. Khan (2000), "Monetary Regimes and Inflation Targeting", *Finance and Development - A quarterly magazine of the IMF*, Vol. 37, No. 3, September, pp. 48-51.
- [29] Danziger, Leif (1988), "Costs of Price Adjustment and the Welfare Economics of Inflation and Disinflation", *American Economic Review*, Vol. 78, No. 4, September, pp. 633-646.
- [30] Debelle, Guy and Stanly Fisher (1994), "How Independent Should a Central Bank Be?", *Federal Reserve Bank of Boston Conference*, Vol. 38.

- [31] Debelle, Guy (1999), "Inflation Targeting and Output Stabilization", Reserve Bank of Australia Research Discussion Paper, No 8.
- [32] Debelle, Guy, Paul Masson, Miguel Savastano, and Sunil Sharma (1999), "Inflation Targeting as a Framework for Monetary Policy", *Economic Issues*, IMF, September, No. 15.
- [33] *Economic Report of the President* (1990), Washington, D.C, U.S. Government Printing Office.
- [34] Erceg, Christopher J. and Andrew T. Levin (2003), "Imperfect Credibility and Inflation Persistence", *Journal of Monetary Economics*, Vol. 50, Issue 4, pp. 915-944.
- [35] Fackler, Paul W. and Mario J. Miranda (2001), "Solving Non-Linear Rational Expectations Models", Working Paper, Ohio State University.
- [36] Fair, Ray and John B. Taylor (1983), "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models", *Econometrica*, Vol. 51, Issue 4, pp. 1169-85.
- [37] Fisher, P.G. (1992), *Rational Expectations in Macroeconomic Models*, Kluwer Academic Publisher, Dordrecht, Netherlands.
- [38] Fischer, Stanley (1996), "Why Are Central Banks Pursuing Long-Run Price Stability", Federal Reserve Bank of Kansas City.
- [39] Forder, James (2004), "'Credibility' in Context: Do Central Bankers and Economists Interpret the Term Differently?", *Econ Journal Watch*, Vol. 1, No. 3, pp. 413-426.
- [40] Friedman, Benjamin M. and Kenneth Kuttner (1996), "A Price Target for US Monetary Policy? Lessons from the Experience with Money Growth Targets", *Brookings Papers on Economic Activity*, No. 1, pp. 77-125.
- [41] Fuhrer, Jeff and George Moore (1995), "Inflation Persistence", *Quarterly Journal of Economics*, Vol. 110, No. 1, pp. 127-159.

- [42] Galí, Jordi (2000), "New Perspectives on Monetary Policy, Inflation, and the Business Cycle", *NBER Working Paper*, No. 8767.
- [43] Goodhart, Charles and Jose Vinals (1994), "Strategy and Tactics of Monetary Policy: Examples from Europe and the Antipodes", in *Goals, Guidelines, and Constraints Facing Monetary Policymakers*, edited by Jeffrey Fuhrer, Federal Reserve Bank of Boston.
- [44] Gordon, Robert J. (1982), "Why Stopping Inflation May Be Costly: Evidence from Fourteen Historical Episodes" in *Inflation: Causes and Effects*, edited by Robert E. Hall, University of Chicago Press, Chicago, pp. 11-40.
- [45] Grenville, Stephen (2000), "Inflation Targeting in a World of Volatile Capital Flows", paper prepared for the Bank of Thailand's International Economic Conference, *Practical Experiences on Inflation Targeting*, Bangkok.
- [46] Haldane, Andrew (1995), *Targeting Inflation: A Conference of Central Banks on the use of Inflation Targets* organised by the Bank of England, London, 9-10 March, Imprint London : Bank of England.
- [47] Haldane, Andrew (1997), "Some Issues in Inflation Targeting", *Bank of England Working Paper*, No 74.
- [48] Huh, Chan G. and Kevin J. Lansing (2000), "Expectations, Credibility and Disinflation in a Small Macroeconomic Model", *Journal of Economics and Business*, Vol. 51, pp. 51-86.
- [49] Ireland, Peter (1995), "Optimal Disinflation Path", *Journal of Economic Dynamics and Control*, Vol. 19, pp. 1429-1448.
- [50] Ireland, Peter N. (1997), "Stopping Inflation, Big and Small", *Journal of Money, Credit and Banking*, Vol. 29, No.4 (November, Part 2), pp. 759-775.
- [51] Judd, Kenneth (1998), *Numerical Methods in Economics*, MIT Press.
- [52] Keynes, John Maynard (1923), *A Tract on Monetary Reform*, MacMillan.
- [53] Khan, Aubik, Robert G. King and Alexander L. Wolman (2003), "Optimal Monetary Policy", *Review of Economic Studies*, Vol. 70, Issue 4, pp. 825-860.

- [54] King, Robert G. and Alexander L. Wolman (1999), "What Should the Monetary Authority Do When Prices are Sticky?" in *Monetary Policy Rules*, edited by John Taylor, University of Chicago Press for NBER, Chicago pp. 349-404.
- [55] King, Mervyn (1996), "How Should Central Banks Reduce Inflation? - Conceptual Issues", in *Achieving Price Stability*, Kansas City: Federal Reserve Bank of Kansas.
- [56] King, Mervyn (1999), "Challenges for Monetary Policy: New and Old", in *New Challenges for Monetary Policy*, A Symposium Sponsored by The Federal Reserve Bank of Kansas City.
- [57] Kydland, Finn and Edward Prescott (1977), "Rules Rather than Discretion: The Inconsistency of Optimal Plans", *Journal of Political Economy*, Vol. 85, pp. 473-92.
- [58] Leiderman, Leonardo and Lars E.O. Svensson (1995), "Inflation Targets", Center of Economic Policy Research, London.
- [59] Longworth, David and Brian O'Reilly (2000), "The monetary Policy Transmission Mechanism and Policy Rules in Canada", Central Bank of Chile Working Paper, No. 72.
- [60] Lucas, Robert E. Jr. (1976), "Econometric Policy Evaluation: A Critique", *Carnegie-Rochester Conference Series on Public Policy*, No. 1, pp. 19-46.
- [61] Lucas, Robert E. Jr. and Nancy L. Stockey (1983), "Optimal Fiscal and Monetary Policy in an Economy without Capital", *Journal of Monetary Economics*, Volume 12, Issue 1, pp. 55-93.
- [62] Lucas, John (2000), "Inflation and Welfare", *Econometrica*, Vol. 68, No. 3, pp. 247-274.
- [63] Marimon, Ramon and Andrew Scott (1999), *Computational Methods for the Study of Dynamic Economies*, Oxford University Press.
- [64] Masson, Paul, Miguel Savastano, and Sunil Sharma (1997), "The Scope for Inflation Targeting in Developing Countries", *IMF Working Paper*, No. 130.

- [65] Masson, Paul R., Miguel A. Savastano and Sunil Sharma (1998), "Can Inflation Targeting Be a Framework for Monetary Policy in Developing Countries?", *Finance and Development - A quarterly magazine of the IMF*, Vol. 35, No. 1, March, pp. 34-37.
- [66] Masson, Paul R. (1999), "Monetary and Exchange Rate Policy of Transition Economies of Central and Eastern Europe after the Launch of EMU", in BLEJER, Mario & SKREB, Marko (Eds.) *Central Banking, Monetary Policies, and the Implications for Transition Economies*, Dordrecht, Kluwer, pp. 429-452.
- [67] McCallum, Bennett T. (1987), "The Case for Rules in the Conduct of Monetary Policy: A Concrete Example", *Federal Reserve Bank of Richmond Economic Review*, Vol. 73.
- [68] McCallum, Bennett T. (1993), "Specification and Analysis of a Monetary Policy Rule for Japan", Bank of Japan Monetary and Economic Studies, Vol. 11.
- [69] McCallum Bennett T. (1995) , "Rules for Monetary Policy", NBER Reporter, Research Summaries, Spring 1995, pp. 5-8.
- [70] McCallum Bennett T. (1996), "Inflation Targeting in Canada, New Zealand, Sweden, the United Kingdom, and in General", *NBER Working Paper*, No. 5579.
- [71] McCalum, Bennett T. and Edward Nelson (1999), "Nominal Income Targeting in an Open-Economy Optimising Model", *Journal of Monetary Economics*, Volume 43, Issue 3, pp. 553-578.
- [72] McCallum Bennett T. (1999), "Recent Developments in Monetary Policy Analysis: The Roles of Theory and Evidence", *NBER Working Paper*, No 7088.
- [73] McCallum, Bennett T. (2000), "Alternative Monetary Policy Rules: A Comparison with Historical Settings for the United States, the United Kingdom and Japan", *Federal Reserve Bank of Richmond Economic Quarterly*, Vol. 86, pp. 49-78.
- [74] Meltzer, Allan H. (1999), "A Policy for Japanese Recovery," working paper.

- [75] Mishkin, Frederic S. (2000), "Inflation Targeting in Emerging Countries", *NBER Working Paper*, No. 7618.
- [76] Mishkin, Frederic S. and Klaus Schmidt-Hebbel (2000), "One Decade Of Inflation Targeting In The World: What Do We Know And What Do We Need To Know?" Central Bank of Chile Working Paper, No. 101.
- [77] Mishkin, Frederic S. (2004), "Can Central Bank Transparency Go Too Far?" *NBER Working Paper*, No. W10829.
- [78] Nelson, Edward (2000), "UK Monetary Policy 1972-97: A Guide Using Taylor Rules", *Bank of England Working Paper*, No 120.
- [79] Nicolae, Anamaria and Charles Nolan, "The Impact of Imperfect Credibility in a Transition to Price Stability", *Journal of Money, Credit and Banking*, forthcoming.
- [80] Orphanides, Athanasios (1999), "The Quest for Prosperity without Inflation", ECB Working Paper, No. 15.
- [81] Pauletto, Giorgio (1997), *Computational Solution of Large-Scale Macroeconomic Models*, Kluwer Academic Publisher, Dordrecht, Netherlands, also <http://www.unige.ch/ses/metri/pauletto/t.ps>.
- [82] Phelps, Edmund S. (1979), "Disinflation Without Recession: Adaptive Guideposts and Monetary Policy", in *Studies in Macroeconomic Theory*, New York, Academic Press.
- [83] Rogoff, Kenneth (1985), "The Optimal Degree of Commitment to an Intermediate Monetary Target", *Quarterly Journal of Economics*, Vol. 100, No. 4, pp. 1169-1190.
- [84] Rotemberg, Julio J. and Michael Woodford (1992), "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity", *Journal of Political Economy*, Vol. 100, December, pp. 1153-1207.
- [85] Sargent, Thomas J. (1982), "The Ends of Four Big Inflations" in *Inflation: Cause and Effect*, edited by Robert E. Hall, Chicago University Press, Chicago, pp. 41-97.

- [86] Schaechter Andrea, Mark R. Stone and Mark Zelmer (2000), "Adopting Inflation Targeting: Practical Issues for Emerging Market Countries", *IMF Occasional Paper*, No 202.
- [87] Srour, Gabriel (1999), "Inflation Targeting under Uncertainty", Bank of Canada Technical Report, No. 85.
- [88] Srour, Gabriel (2003), "Some Notes on Monetary Policy Rules with Uncertainty", Bank of Canada Working Paper, No. 16.
- [89] Svenson, Lars E. O (1996), "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets", *Bank of England Working Paper*, No 56.
- [90] Svensson, Lars E.O. (1997), "Inflation Targeting: Some Extensions", *NBER Working Paper*, No. 5962.
- [91] Svensson, Lars E.O. (1998), "Open-Economy Inflation Targeting", *NBER Working Paper*, No. 6545.
- [92] Svensson, Lars E.O. (2000), "How Should Monetary Policy be Conducted in an Era of Price Stability?", *NBER Working Papers*, No. 7516.
- [93] Svensson, Lars E. O. (2003), "Monetary Policy and Real Stabilization", *NBER Working Paper*, No. 9486.
- [94] Taylor, John B. (1983), "Union Wage Settlements during a Disinflation", *American Economic Review*, Vol. 73, No. 5, pp. 981-983.
- [95] Taylor, John B. and Harald Uhlig (1990), "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods", *Journal of Business and Economic Statistics*, Vol. 8, No. 1, pp. 1-18.
- [96] Taylor, John B. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, pp. 195-214.
- [97] Taylor, John B. (1998), "Applying Academic Research on Monetary Policy Rules: An Exercises in Translational Economics", Presented at the Macro, Money, and Finance Research Group Conference, Durham University, September 12, 1997.



- [98] Taylor, John B. (1999), *Monetary Policy Rules*, University of Chicago Press, Chicago.
- [99] Taylor, John B. (2000), "Using Monetary Policy Rules in Emerging Market Economies", in *Stabilization and Monetary Policy: The International Experience*, Bank of Mexico, pp. 441-457.
- [100] Yates, Anthony (1998), "Downward Nominal Rigidity and Monetary Policy", *Bank of England Working Paper*, No. 82.
- [101] Woodford, Michael (1999a), "Optimal Monetary Policy Inertia", *NBER Working Paper*, No. 7261.
- [102] Woodford, Michael (1999b), "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?" in *New Challenges for Monetary Policy*, Kansas City: Federal Reserve Bank of Kansas City.
- [103] Woodford, Michael (2003), "Price-Level Determination under Interest-Rate Rules", Chapter 2 in *Interest and Prices*, Princeton University Press.

